Essays on the Macroeconomics of Labor Markets

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José-Víctor Ríos-Rull

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Abstract

This thesis is composed of three chapters that study the behavior of economies where trade in both labor and goods market is subject to search frictions. It analyzes the interactions between frictional labor and goods markets and examines technology and preference shocks as alternative sources of business cycle fluctuations in unemployment, hours worked and inventories.

In Chapter 1, the focus is on the Diamond-Mortensen-Pissarides model with Nash wage bargaining. This model provides a qualitatively appealing theory of unemployment, but its ability to explain the observed magnitude of fluctuations in unemployment remains debated. I add goods market frictions to this model, and show that they affect workers' bargaining position, provide a rationale for a high value of non-market activity and also affect its cyclical properties. These frictions can thus amplify the response of unemployment and vacancies to changes in the measured labor productivity caused by either technology or preference shocks. The response of the vacancy-unemployment ratio in the extended model is about twice as large as in the model with labor search only if either (1) goods and search effort are substitutes in the goods market matching function and fluctuations are a result of a technology shocks, or (2) when goods and search effort are complements in the goods market matching function and the driving force of fluctuations are preference shocks. Finally, I show that if preferences are additively separable and goods market matching function has unit elasticity of substitution, preference and technology shocks are observationally equivalent and can not be separately identified by an economist who would analyze data on labor productivity, output, employment and wages.

Chapter 2 shows that introducing goods market frictions into an otherwise standard model provides a simple but attractive framework to analyze the behavior of inventories over the business cycle. It also shows that the behavior of sales and inventories over the business cycle contains information that allows to identify the contribution of technology and preference shocks to fluctuations in unemployment. I employ Bayesian methods to estimate a model with goods and labor search frictions using U.S. data on labor productivity and inventory-sales ratio, and find that the implied response of

vacancies and unemployment to changes in measured labor productivity is about twice as large as in the model with labor search only. Goods market frictions also allow the model to account for the main facts on inventories - procyclical inventory investment, countercyclical inventories-sales ratio, and sales which are more volatile than production.

In Chapter 3, I examine another shortcoming of the labor search model identified by Shimer (2010) and related to the so called labor wedge - the gap between the firm's marginal product of labor and the household's marginal rate of substitution between consumption and leisure. I show that under the business cycle accounting approach proposed by Chari, Kehoe, and McGrattan (2007) goods market frictions in the model manifest themselves as a labor wedge: In an expansion, firms find it easier to sell goods, and consumers benefit from higher availability of goods and smaller disutility from search effort required per unit of consumption purchased; this encourages larger response of the intensive margin of labor supply than in the standard frictionless model. It thus also alleviates the issue arising in model with frictional labor markets, where search frictions act as adjustment costs, and thus result in a labor wedge that resembles a counterfactually procyclical tax on labor income.

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Chapter 1

Amplification of Shocks in a Model with Goods and Labor Search

1.1 Introduction

It is widely accepted that heterogeneities and information imperfections make trade in the labor market a decentralized, time consuming and costly activity for firms and workers. Similar complications arise with trade in the goods market. With heterogeneity in characteristics of goods and services, and with costly acquisition of information, consumers have to spend resources to find the goods and services that match their needs and preferences, and to obtain information about their availability at different locations. But while the literature studying departures from the Walrasian labor market by imposing search frictions is quite large (see Pissarides, 2000 for introduction to the literature, and Rogerson, Shimer, & Wright, 2005 for a survey), similar analysis for the goods market is less common.

The aim of this paper is to study how unemployment dynamics is affected by effects that arise from interactions of frictional labor and goods markets. To that end, I extend the standard Diamond-Mortensen-Pissarides labor search-matching model by introducing a goods market search-matching friction, and use it to address two issues.

First, the response of unemployment to changes in labor productivity in the basic labor search model is much smaller than in U.S. data; I show that feedback effects between labor market and goods market can result in amplification of shocks in the extended model. Higher employment increases output which can encourage consumers to increase their search for consumption goods; higher search effort by consumers increases profits of firms and thus affects firms' hiring decisions. Moreover, when wages are determined by Nash bargaining, there is an additional effect through the wage channel. In the extended model, goods market frictions affect worker's bargaining position, provide rationale for high value of non-market activity, but also change cyclical properties of the value of non-market activity. This effect arises since higher availability of goods in expansions makes frictions in the goods market less severe from consumer's perspective, thus increasing the value of additional earnings obtained when the worker accepts the job. This results in a downward pressure on worker's outside option in the Nash bargaining and consequently increases incentives for firms to hire new workers.

Second, I examine the driving forces behind unemployment fluctuations, and in addition to technology (supply side) shocks also consider preference (demand side) shocks, that give rise to movements in measured labor productivity similar to those caused by technology shocks. In the model with goods market frictions, a demand shock that increases the search effort by consumers also increases output and measured labor productivity. These shocks can therefore provide an alternative explanation of fluctuations in unemployment over the business cycle. I show that to an economist who would use only the time series usually considered in labor search literature - labor productivity, output, employment, vacancies and wages - preference and technology shocks are observationally equivalent if utility is additively separable and at the same time goods market matching function has unit elasticity of substitution. This means that based on these time series it is impossible to distinguish the case with actual shocks to technology from the case where the true productivity is constant, and changes in measured average labor productivity, output and employment are the result of changes in preferences and demand.

I first explore the qualitative properties of the model, analyze conditions under which technology and preference shocks can be distinguished, and under which goods market frictions amplify effects of shocks. After that, I estimate the model using likelihood based Bayesian methods; this approach is used since in the presence of goods market frictions measured labor productivity becomes endogenous and does not coincide with actual unobserved productivity. The model is estimated with one shock at a time, to match only the time series for U.S. average labor productivity. When search effort and output supplied by firms are good substitutes, a modest amount of goods market frictions increases the response of vacancy-unemployment ratio to technology shocks by one third. And with low substitutability between search effort and output supplied by firms, shocks to preferences result in response of vacancy-unemployment ratio which is about two and half times larger than the response to technology shocks in model with labor search only.

The rest of the paper is organized as follows. After the review of related literature, the model is described in Section 1.2, next, equilibrium is characterized and its qualitative properties are examined in Section 1.3. In Section 1.4 I use Bayesian estimation of a weekly model matching the U.S. labor productivity to parametrize alternative shocks, and then compare the implied business cycle properties of unemployment, vacancies and the labor market tightness. Section 1.5 concludes. Most of the algebra used to derive the characterization of equilibrium, as well as all the proofs are delegated to the appendices.

Related Literature

The ability of the Diamond-Mortensen-Pissarides search and matching model of the labor market (Diamond, 1982, Pissarides, 1985, Mortensen & Pissarides, 1994, and also Pissarides, 2000 for textbook exposition) to amplify and propagate the technology shocks and the extent to which model generated business cycles statistics match the U.S. data have been widely discussed. Shimer (2005) argues that the basic model calibrated to U.S. data can not generate enough volatility in unemployment, vacancies and in labor market tightness: while surplus of a match increases in expansions, under Nash bargaining wages absorb most of this increase, leaving firms with little incentives to hire new workers. Several papers thus examined different examined wage rigidity (Shimer, 2005, Hall, 2005) and alternative wage bargaining process (Hall & Milgrom, 2008, Mortensen & Nagypal, 2007) as a ways to improve the performance of the model. The wage rigidity required is that wages of workers in new employment relationships are rigid over the business cycle. Given that the empirical evidence available does not support this claim (see Pissarides, 2009 for a detailed discussion), this solutions is not completely without its own problems.

After investigating the puzzle more closely, Hagedorn and Manovskii (2008) have proposed an alternative way to calibrate the two key parameters - worker's bargaining power and the value of the non-market activity - and were able to obtain fluctuations of the right magnitude. However, as shown in Hornstein, Krusell, and Violante (2005) and Costain and Reiter (2008), with this alternative calibration the response of unemployment to changes in unemployment compensation in the model is implausibly large. Several other papers examined modifications of the basic labor search model to see if they improve its quantitative properties; these include among others labor turnover costs (Mortensen & Nagypal, 2007, Pissarides, 2009, Silva & Toledo, 2013), asymmetric information (Guerrieri, 2008, Moen & Rosén, 2011), endogenous home production (Garin & Lester, 2013), and introduction of on-the-job search (Krause & Lubik, 2010, Menzio & Shi, 2011). In all these papers changes in productivity as a result of technology shocks remain the driving source of business cycle fluctuations.

A promising alternative to technology shocks was proposed by Bai, Ríos-Rull, and Storesletten (2012). These authors show that once goods market frictions are incorporated into a traditional RBC model with frictionless labor market, preference shocks generate movements in Solow residual similar to those caused by technology shocks, and also perform well in matching co-movements of main macroeconomic variables. Their results thus motivate to consider preference shocks as an alternative to the technology shocks in the model with labor search. But since labor market is frictionless in their model, the results of interaction of frictions in labor and goods markets are not investigated in their paper.

There are a few papers that lately started to analyze the interactions of search frictions in labor and goods markets. Lehmann and Van der Linden (2010) investigate the link between inflation and unemployment in a modified labor search model where products are sold in frictional market with demand given by real money holdings of consumers. Kaplan and Menzio (2013) develop a model where shopping externalities lead to multiplicity of equilibria, and where shocks to agents' expectations about future unemployment create self-fulfilling fluctuations even in absence of any shocks to technology or preferences. Huo and Ríos-Rull (2013) develop a neoclassical growth model with tradable and nontradable sector, frictions in goods and labor markets, and with adjustment costs in both physical investment and hiring of new employees. Goods market search frictions exist at the level of varieties household consumes rather than firms' locations as in Bai et al. (2012), search effort is a complement rather than a substitute for the resources spent, and preferences with no wealth effects guarantee that varieties of nontradable goods are a normal good. The paper analyzes the effects of wealth and financial shocks instead of traditionally considered shocks to total factor productivity, and show how the increased desire to save by consumers can induce a recession rather than a boom. This recession arises due to the adjustment cost and labor market frictions; goods market frictions are important quantitatively and amplify the recession. The focus of my paper is different, I examine technology and the preference shocks as alternative source of business cycle fluctuations in the Diamond-Mortensen-Pissarides model with goods market frictions, and study the channels through which search frictions in the goods market amplify the response of unemployment to changes in measured labor productivity.

The two papers that are probably closest to mine are Petrosky-Nadeau and Wasmer (2011) and Michaillat and Saez (2013). Petrosky-Nadeau and Wasmer (2011) consider the standard technology shocks only, and show that in a model with search in credit, labor and goods markets technology shocks are both significantly amplified and propagated by goods market frictions. Their framework is different from the one this paper. Firms and consumers in the goods market form long term matches and price for which output is sold in these matches is determined by bilateral Nash bargaining. Matching frictions in the good market in their model introduce a delay in the reaction of unemployment to technology shocks through firms' response to the evolution of price and congestion in the goods market, but with linear preferences and a simple wage setting rule their approach misses the effects of goods market frictions on outside option of the worker in the wage bargaining process. Moreover, the fact that the technology shock process is parameterized the same way in their models with and without goods market frictions implies that the properties of the measured labor productivity will be different in these models.

The paper by Michaillat and Saez (2013) analyzes the role of demand and supply shocks in shaping the aggregate demand and employment when labor and goods markets are subject to search frictions. In addition, they study the size of the government purchase multiplier and effects of redistributional transfers and changes in minimum wage on output and employment. The focus of their paper is however on theoretical analysis of the short run, and the model they develop is static, with prices that are fixed. If prices and wages were instead determined by Nash bargaining, demand shocks would have no effect on labor market tightness. In contrast, prices in my model are flexible, amplification effects are not driven by price or wage rigidities, and demand shocks play an important role in explaining fluctuations in vacancies and unemployment.

1.2 Model

There is a measure one of households, each with measure one of infinitely lived workers. Workers have to search for jobs in the labor market, and search for consumption goods in the goods market. Household pools resources and provides its members insurance against fluctuations that arise due to the uncertain results of search. Preferences are subject to shocks affecting the marginal utility of consumption and marginal disutilities from work and search. These shocks are perfectly correlated across all workers in the economy.

There is also a continuum of firms with measure one which use labor as the only input to produce goods. Goods are sold in market that is subject to search frictions, firms post prices and consumers direct their search effort to acquire goods at a particular price. I assume that workers cannot quit but there is exogenous job destruction. Firms need to open and maintain vacancies to hire new workers. For labor market I employ standard undirected search mechanism with Nash bargaining.

The aggregate state of the economy is $\mathbf{S} = (z, \zeta, N)$, where N is the measure of employed workers after separations take place and (z, ζ) are the exogenous shocks with z being the current technology, ζ the current preference shock. I assume that shocks (z, ζ) follow first order Markov process.

Time is discrete and the timing of events within the period is as follows: (1) shocks are realized; exogenous job separations occur; (2) each firm decides simultaneously how many vacancies to open and the price for which to sell goods; (3) employed workers produce, unemployed workers search for a job, search and matching in the goods and labor markets takes place; (4) payments are made (goods purchases, dividends, wages); (5) household pools resources and goods purchased, consumption takes place.

Labor Market

As in the basic labor search-matching model in Pissarides (2000), only unemployed workers search for jobs, search is not directed, and the number of matches of unemployed workers U and vacancies V is given by an aggregate constant returns to scale matching function $m^L(U,V)$. Let $\theta = \frac{V}{U}$ denote the tightness of the market, $\pi^u(\theta) = m^L(1,\theta)$ the probability for an unemployed worker to be hired, and $\pi^v(\theta) = m^L(1/\theta, 1)$ the measure of workers that one vacancy attracts.

I assume that workers value their actions based on the contribution they bring to the utility of the household; worker's surplus from being employed is thus the change in the household's utility from having one additional member employed. When a worker and a vacancy are matched, and the worker accepts the job, wage w is set in every period as a solution to the asymmetric Nash bargaining problem[1]

$$w(\mathbf{S}) = \underset{\hat{w}}{\operatorname{argmax}}\, \hat{W}_n(\hat{w})^{\mu} \hat{\Omega}_n(\hat{w})^{1-\mu} \tag{1.2.1}$$

where $\hat{W}_n(\hat{w})$ and $\hat{\Omega}_n(\hat{w})$ are the household's and firm's value of a marginal worker employed and earning arbitrary wage $\hat{w}$ in the current period and equilibrium wage w thereafter, until the job is hit by the separation shock δ.

Goods Market

Acquisition of consumption goods requires active search effort on the side of the consumer to find the goods to purchase. To model these frictions in the goods market I adopt the competitive search mechanism proposed by Moen (1997) - firms post prices and consumers direct their search effort to acquire goods at a particular price. Goods market is thus divided into submarkets, and firm and household's members can choose in which submarket to trade. The amount of goods sold in any submarket is determined by

[1] The timing of payments is however not crucial. Even if wages are constant throughout the duration of employment, as long as at the time when a match is formed the surplus is split according to the Nash bargaining, firms' decisions about hiring are unaffected. Equilibrium allocation is then same as in the case with period by period Nash bargaining.

a matching function $m^G(D, TX)$. Here D is the aggregate search effort of all consumers in the particular submarket, T the measure of firms selling in the particular submarket and X is the quantity of goods sold per firm in the submarket.

Assumption 1. *Goods market matching function $m^G(D, TX)$ is constant return to scale, with elasticity of substitution σ.*

Submarkets are indexed by (p, Q) where p is the price of the consumption good and $Q = \frac{T}{D}$ is the tightness of the submarket. Since m^G has constant returns to scale, the amount of goods acquired per unit of search effort by household's shopper is

$$\psi^d(Q, X) = m^G(1, QX)$$

and the probability that a particular unit of good is sold is

$$\psi^x(Q, X) = m^G\Big(\frac{1}{QX}, 1\Big)$$

Consequently, the amount of output successfully sold by a firm supplying x in submarket (p, Q), where the total amount of goods supplied by all firms is TX is

$$x\psi^x(Q, X) = \frac{x}{X}\frac{\psi^d(Q, X)}{Q}$$

The matching process is thus different from the one in Bai et al. (2012). Here, ceteris paribus, an increase in the total supply of goods in the submarket affects the probability that a particular unit of consumption good is sold, whereas in their paper that probability is unaffected. This modified assumption seems intuitive, and as discussed in Section 1.3 it allows to identify the relative importance of technology and preference shocks. In addition, efficiency in Bai et al. (2012) requires stronger assumption on information available to consumers - equilibrium in is guaranteed to be efficient only if submarket are indexed by price, tightness and also quantity sold; as shown below, only price and tightness are needed in my model.

Households

As in Merz (1995), I consider households to be extended families consisting of a measure one of workers. All workers are ex-ante identical and their preferences are given by

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t \mathrm{u}(c_t, d_t, e_t, \zeta_t)$$

where c_t is consumption, d_t search effort in goods market, e_t is the employment status and $\zeta_t = (\zeta_{ct}, \zeta_{dt}, \zeta_{nt})$ is the preference shock affecting the marginal utility of consumption, marginal disutility of search for consumption good and the disutility of work.

Households own firms, and in the recursive formulation of the household's problem, the individual state of a household, $\mathbf{s} = (a, n)$, is given by wealth in the form of shares a, and the number of members of the household that have a job after separations take place n. Household decides about goods market search effort of its employed and unemployed workers d^n, d^u, consumption allocation c^n, c^u, and about share holdings for next period a'. Each member also decides in which submarket (p, Q) to search for consumption goods, and directs the search to the submarket that delivers the biggest contribution to the utility of the household. I incorporate this through a constraint in the problem of a firm which posts price and decides about quantity sold. In addition, since in equilibrium only one market is going to be active, in the household's problem price of goods, goods market tightness and quantity sold appear as given functions of state $p(\mathbf{S}), Q(\mathbf{S}), X(\mathbf{S})$.

Taking prices $p(\mathbf{S})$, $w(\mathbf{S})$, $R(\mathbf{S})$ as given, the household then faces a budget constraint

$$p(\mathbf{S})\big(nc^n + (1-n)c^u\big) + a' = (1 + R(\mathbf{S}))a + w(\mathbf{S})n$$

with shares acting as a numeraire. In addition, search frictions in goods market impose a constraint

$$nc^n + (1-n)c^u = \big(nd^n + (1-n)d^u\big)\psi^d(Q(\mathbf{S}), X(\mathbf{S}))$$

where $\psi^d(Q, X)$ is the amount of goods acquired per unit of search effort in the goods markets, and the search frictions in labor market constraint

$$n' = (1-\delta)n + \pi^u(\theta(\mathbf{S}))(1-n)$$

where $\pi^u(\theta)$ is the probability for an individual to find a job. Since the optimal allocation of consumption and search effort among family members in each period solves

$$
\begin{aligned}
&U(c,d,n,\zeta) = \max_{c^n,c^u,d^n,d^u} n\mathrm{u}(c^n,d^n,1,\zeta) + (1-n)\mathrm{u}(c^u,d^u,0,\zeta)\\
&\text{subject to}\\
&nc^n + (1-n)c^u = c\\
&nd^n + (1-n)d^u = d
\end{aligned}
$$

where c is the total amount of consumption goods available to household and d is the overall search effort, I can formally set up the household's problem in which it acts as if it had preferences with utility function $U(c,d,n,\zeta)$. To summarize, household's problem written in a recursive form is

$$
\begin{aligned}
&W(\mathbf{s};\mathbf{S}) = \max_{c,d,a'} U(c,d,n,\zeta) + \beta\mathbb{E}W(\mathbf{s}';\mathbf{S}') \qquad (1.2.2)\\
&\text{subject to}\\
&p(\mathbf{S})c + a' = (1+R(\mathbf{S}))a + w(\mathbf{S})n\\
&c = d\psi^d(Q(\mathbf{S}),X(\mathbf{S}))\\
&n' = (1-\delta)n + \pi^u(\theta(\mathbf{S}))(1-n)\\
&\mathbf{S}' = G(\mathbf{S})
\end{aligned}
$$

Firms

The individual state of a firm is the number of workers employed n. Each firm chooses in which submarket (p,Q) to sell the goods and at the same time decides how many vacancies v to open. The amount of goods x that the firm can potentially sell is given by

$$x = zf(n-\chi v) - \kappa(v)$$

with $f_l > 0$, $f_{ll} \leq 0$ and $\kappa_v \geq 0$, $\kappa_{vv} \geq 0$ which can be interpreted as a case where some of the workers act as recruiters and thus χv hours of worked are diverted from the production process to hiring, and in addition $\kappa(v)$ costs in terms of goods are incurred for vacancy posting. This specification of the hiring process nests Shimer (2010) as a special case where $f(l) = l$, $\chi = 1$ and $\kappa(v) \equiv 0$, and the benchmark case from Pissarides (2000) if $f(l) = l$, $\chi = 0$ and $\kappa(v) \equiv \kappa$. Each vacancy attracts $\pi^v(\theta)$ new workers. If the firm decides to sell its output x in the (p, Q) submarket, where the aggregate amount of goods being sold is X, then the actual amount of goods sold is given by

$$x\psi^x(Q, X) = \frac{x}{X}\frac{\psi^d(Q, X)}{Q}$$

Goods which are not sold can not be stored, as in Bai et al. (2012) and Petrosky-Nadeau and Wasmer (2011). As discussed above, the firm needs to take into account a constraint which guarantees shoppers in the (p, Q) submarket equilibrium value of search $W_d^*(\mathbf{S})$. Let $M(\mathbf{S})$ be the marginal value of wealth in terms of utility, then

$$W_d(\mathbf{S}) = U_d(\mathbf{S}) + (U_c(\mathbf{S}) - pM(\mathbf{S}))\psi^d(Q, X)$$

is the value to the household of the marginal search effort in the (p, Q) submarket. Finally, let $m(\mathbf{S}, \mathbf{S}')$ be the stochastic discount factor used to discount future profits. To summarize, the problem that a firm solves is then

$$\Omega(n; \mathbf{S}) = \max_{v,p,Q,x} \left\{p\psi^x(Q, X)x - w(\mathbf{S})n + \mathbb{E}[m(\mathbf{S}, \mathbf{S}')\Omega(n'; \mathbf{S}')]\right\} \tag{1.2.3}$$

subject to

$$x = zf(n - \chi v) - \kappa(v)$$

$$n' = (1 - \delta)n + \pi^v(\theta(\mathbf{S}))v$$

$$W_d^*(\mathbf{S}) = U_d(\mathbf{S}) + (U_c(\mathbf{S}) - pM(\mathbf{S}))\psi^d(Q, X)$$

$$\mathbf{S}' = G(\mathbf{S})$$

Equilibrium

Definition 1. *Equilibrium is household's value function and decision rules $(W, g^c, g^d, g^{a'})$ as functions of $(\mathbf{s}; \mathbf{S})$; value function and decision rules (Ω, g^v, g^p) as functions of $(n; \mathbf{S})$; aggregate allocation (X, C, D, V), tightness (Q, θ), prices (p, w), dividends R, law of motion for employment G^N, all as functions of $\mathbf{S}$; such that*

1. *Value function W solves (1.2.2) and $(g^c, g^d, g^{s'})$ are the associated policy functions*
2. *Value function Ω solves (1.2.3) and (g^v, g^p) are the associated policy functions*
3. *Household and firm are representative*
4. *Wage w solves the Nash bargaining problem (1.2.1)*
5. *Goods market tightness is $Q(\mathbf{S}) = \frac{1}{D(\mathbf{S})}$; labor market tightness $\theta(\mathbf{S}) = \frac{V(\mathbf{S})}{1-N}$*
6. *Law of motion for employment is implied by firm's policy.*

1.3 Characterization of Equilibrium

In this section I analyze the qualitative properties of the model economy, role of technology and preference shocks, and the interactions of frictions in labor market and goods markets. I start by deriving two functional equations that characterize the dynamics of market tightnesses $Q(\mathbf{S})$ and $\theta(\mathbf{S})$. These are obtained by first deriving the optimality conditions for the household and the firm, and then using them to obtain the solution for the Nash bargaining problem in the labor market, and competitive search problem in the goods market. Details can be found in Appendix A and Appendix B, here I summarize the results. To avoid the notational clutter in what follows I drop the arguments of functions, use g_A to denote derivative of function g with respect to A, and g' to denote value of function g in the next period. Thus for example in equation (1.3.1) below U_c and U_d stand for $\frac{\partial}{\partial c}U(C(\mathbf{S}), D(\mathbf{S}), N)$ and $\frac{\partial}{\partial d}U(C(\mathbf{S}), D(\mathbf{S}), N)$, ψ^d for $\psi^d(Q(\mathbf{S}), X(\mathbf{S}))$. I also use notation ϵ_B^A for elasticity of A with respect to B.

The goods market with competitive search gives rise to the following intratemporal condition

$$-U_d = (1 - \epsilon_Q^{\psi^d})\psi^d U_c \tag{1.3.1}$$

where $\epsilon_Q^{\psi^d} = \frac{\partial \log \psi^d}{\partial \log Q}$.

The labor market behavior is characterized by the following condition, which is the counterpart of the stochastic first order difference equation for market tightness θ in the basic Diamond-Mortensen-Pissarides search matching model

$$\begin{aligned}
&\frac{1}{\pi^v}(\chi z f_l + \kappa_v)\psi^x\left(U_c + \frac{U_d}{\psi^d}\right) \\
&= \beta\mathbb{E}\left[\left((1-\mu)z' f_l' + \left(\frac{1-\delta}{(\pi^v)'} - \mu\theta'\right)(\chi z' f_l' + \kappa_v')\right)(\psi^x)'\left(U_c' + \frac{U_d'}{(\psi^d)'}\right) + (1-\mu)U_n'\right]
\end{aligned} \tag{1.3.2}$$

Notice that using

$$\begin{aligned}
C &= \psi^x(Q, X)X \\
D &= 1/Q \\
X &= zf(N - \chi V) - \kappa(V) \\
V &= \theta(1 - N)
\end{aligned}$$

equations (1.3.1) and (1.3.2) can be written in terms of current and next period's (Q, θ, N) and shocks z, ζ only; thus together with the law of motion for labor

$$N' = (1 - \delta)N + \pi^u(\theta)(1 - N) \tag{1.3.3}$$

they fully characterize the dynamics of (Q, θ, N) in equilibrium.

Note also that the measured average labor productivity in this economy is

$$y = \frac{\psi^x X}{N} \tag{1.3.4}$$

and that it is affected by technology shock z, the discrepancy between the number of workers N and the number of workers in production $L = N - \chi V$, and the size of the goods market frictions. In the presence of goods market frictions preference shocks ζ_c and ζ_d have an effect on measured average labor productivity y through their effect on both ψ^x and X. This serves as a motivation in Section 1.4, where I use Bayesian methods to estimate the model and parametrize processes for shocks ζ_c and ζ_d by matching the labor productivity y the model to the labor productivity observed in U.S. data.

1.3.1 Efficiency

The efficient allocation is defined as an allocation chosen by a social planner facing the same search-matching frictions as the participants in the labor and goods markets in the decentralized economy.

Definition 2. *An allocation is efficient if it solves*

$$\mathcal{W}(z, \zeta, N) = \max_{C,D,X,V} \left\{ U(C, D, N, \zeta) + \beta \mathbb{E} \mathcal{W}(z', \zeta', N') \right\}$$

subject to

$$C = m^G(D, TX)$$

$$X = zf(N - \chi V) - \kappa(V)$$

$$N' = (1 - \delta)N + m^L(1 - N, V)$$

Given this definition, the following proposition gives the condition under which the equilibrium of the decentralized economy in this paper is efficient.

Proposition 1 (Efficiency)**.** *If* $\mu = \epsilon_U^{m^L}$ *equilibrium is efficient.*

Thus even with search frictions in the goods market, when this search is competitive and submarkets are indexed by price and market tightness, familiar condition from Hosios (1990) continues to hold, and equilibrium is efficient as long as workers' bargaining power is equal to the elasticity of labor market matching function with respect to unemployment.

1.3.2 The Role of Goods Market Frictions

The channel through which changes in goods market conditions affect the labor market manifests itself in equation (1.3.2) by the terms ψ^x and $\frac{U_d}{\psi^d}$ that affect the cost of hiring an extra worker on the left hand side of equation (1.3.2), and terms $(\psi^x)'$ and $\frac{U_d'}{(\psi^d)'}$ that affect the benefits of hiring this worker on the right hand side of equation (1.3.2). An increase in the expected probability to successfully sell goods $(\psi^x)'$, or a decrease in the disutility from search for goods $\frac{U_d'}{(\psi^d)'}$ raises future benefits of having an extra worker employed in the similar way as an increase in technology z'. Going back one step, the reason why the new terms appear in the labor market condition (1.3.2) is that the presence of search frictions in the goods market changes the surplus of the match between a worker and a firm and affects the wage bargaining process. As a result, as shown in Appendix B, with Nash bargaining real wage in equilibrium is

$$\frac{w}{p} = \mu\psi^x(zf_l + \theta(\chi zf_l + \kappa_v)) - (1-\mu)\frac{U_n}{U_c + \frac{U_d}{\psi^d}} \tag{1.3.5}$$

Similarly to other search-matching models with Nash bargaining, wage is a weighted average of the value of marginal product of a worker enhanced by the vacancy cost savings, and the marginal rate of substitution between leisure and consumption. Compared to a standard model without goods market search, there are however two important differences. First, the value of the marginal product of a worker and the marginal cost of vacancy per worker are multiplied by ψ^x which captures the fact that only a share of goods are actually successfully sold. Second, marginal utility foregone by switching from non-market activity to market activity $-U_n$ is evaluated in terms of consumption goods using $U_c + \frac{U_d}{\psi^d}$ rather than U_c, where $\frac{U_d}{\psi^d}$ captures the disutility from search for consumption good that is needed to be able to spend the extra earned income. This last fact provides rationale for a high value of non-market activity proposed by Hagedorn and Manovskii (2008), as a way to generate fluctuations in labor market tightness in the standard labor search model that would be comparable to those in the data.

Consider now the effects of a positive technology shock z in the economy with goods market frictions. There are several additional channels that affect the wage and the hiring decision of a firm. Since output supplied X increases, the return from search increases for consumers too. Thus, for preferences where the substitution effect dominates the income effect, search effort increases and goods market tightness falls; as a result firms are more likely to sell the goods, which amplifies the impact of initial increase in productivity on return to production. In addition, higher output supplied X and lower goods market tightness Q have opposing effects on disutility from search effort required to purchase the marginal unit of consumption, and thus also on the bargaining position of the worker, wage, and the hiring decision of a firm.

The effects of different shocks can be characterized further if additional assumptions are imposed on preferences and technology. I analyze the behavior of model economy under the assumption of standard separable preferences, and for comparison also under the alternative assumption of preferences for which there is no income effect on search effort.

Assumption 1A. *Utility function of worker is* $\mathrm{u}(c,d,e,\zeta) = \zeta_c u(c) - \zeta_d g(d) - \zeta_n e$ *with relative risk aversion coefficient* $\eta = -\frac{cu''(c)}{u'(c)}$.

Assumption 1B. *Utility function of worker is* $\mathrm{u}(c,d,e,\zeta) = \zeta_c u\big(c - \zeta_d g(d)\big) - \zeta_n e$.

Assumption 2. *Vacancy costs are* $\kappa(v) = z\bar{\kappa}(v)$ *for some* $\bar{\kappa}(v)$ *with* $\frac{d\bar{\kappa}}{dz} = 0$.

Under Assumptions 1A and 2, if in addition the goods market matching function m^G has elasticity of substitution $\sigma = 1$, then preference shocks in this model are in a sense observationally equivalent to technology shocks. That is, a process for technology shock z that generates a particular observed history of average labor productivity y can be replaced by constant technology z and some process for preference shock ζ_d that generates same history y. In addition, observed histories for vacancies, employment, output and wages are also identical. Thus technology shock z and preference shocks ζ_d generate same co-movements of measured labor productivity y and labor market tightness θ.

Proposition 2 (Equivalence of preference and technology shocks)**.**
Under Assumptions 1A and 2

a. iff goods market matching function m^G *has elasticity of substitution* $\sigma = 1$*, then for any history of shocks* (z^t, ζ^t) *and resulting history of average labor productivity, market tightness and employment* $(y^t, \theta^t, Q^t, N^t)$ *which satisfy (1.3.1)-(1.3.4), there exist a history* $(\tilde{z}^t, \tilde{\zeta}^t)$ *and* $\tilde{Q}^t$*, with* $\tilde{z}_t = 1$*,* $\tilde{\zeta}_{ct} = \zeta_{ct}$*,* $\tilde{\zeta}_{nt} = \zeta_{nt}$*, such that* $((\tilde{z}^t, \tilde{\zeta}^t), (y^t, \theta^t, \tilde{Q}^t, N^t))$ *also satisfy (1.3.1)-(1.3.4).*

b. histories of real wages $(\frac{w}{p})^t$ *and* $(\frac{\tilde{w}}{\tilde{p}})^t$ *associated with* $(\tilde{z}^t, \tilde{\zeta}^t)$ *and* $(\tilde{z}^t, \tilde{\zeta}^t)$ *are identical if and only if goods market matching function* m^G *has elasticity of substitution* $\sigma = 1$*.*

This result has implications for the quantitative analysis: assuming additively separable preferences, Cobb-Douglas goods market matching function and vacancy costs proportional to z implies that the two types of shocks can be distinguished, and their contribution to business cycle fluctuations analyzed only if some data on sales relative to the total supply of output in the market is utilized. To an economist who would use only the time series usually considered in labor search literature - labor productivity, output, employment, vacancies and wages - preference and technology shocks are observationally equivalent, it is impossible to distinguish the case with shocks to technology from the case where the actual technology is constant, and changes in measured average labor productivity, output and employment are the results of changes in preferences and demand.

The next proposition establishes a neutrality result for the case where utility function is logarithmic in consumption.

Proposition 3 (Neutrality of shocks)**.**
Under Assumption 1A if in addition $\eta = 1$ *and* $\sigma = 1$

a. technology shocks z *have no effect on the goods market tightness* Q

b. preference shocks ζ_d *have no effect on the labor market tightness* θ

Under Assumptions 1A and 2, if in addition $\eta = 1$ *and* $\sigma = 1$

c. technology shocks z *have no effect on the labor market tightness* θ

In Proposition 3, labor market tightness θ becomes independent of $\{z, \zeta_d\}$ and depends on $\{\zeta_c, \zeta_n\}$ only; goods market tightness Q becomes completely independent of technology z and the behavior of θ in the labor market, and depends on $\{\zeta_c, \zeta_d\}$ only.

The reason for the neutrality result of the goods market with respect to technology z is that the income and substitution effects for search effort in the goods market cancel: For a given level of employment, improvement in technology z results in higher amount of output X supplied by firms and hence allows agents to consume more even if they decrease their search effort, while the substitution effect motivates greater search effort; and if $\eta = 1$ these two effects exactly offset each other. Then, since the search effort is constant so is the goods market tightness and the probability of selling the good. Note that this holds for any form of vacancy cost $\kappa(v)$, Assumption 2 is not necessary for part a. of the Proposition 3 .

The neutrality of labor market tightness with respect to productivity in the labor search model with additively separable logarithmic utility function and with vacancy costs proportional to z is discussed in Shimer (2010), who emphasizes that it holds for any bargaining power of the worker, and any value of non-market activity (leisure) in his model. As we can see, this result holds even in the model with labor and goods search, for *any* amount of frictions in the goods market. That is, it holds for specification of goods matching function as long as m^G has elasticity of substitution $\sigma = 1$. Moreover, labor market tightness θ in this model is also neutral with respect to preference shocks ζ_d. Note that this holds for any form of vacancy cost $\kappa(v)$ since Assumption 2 is not necessary for claim b. of the Proposition 3. Thus the neutrality of labor market tightness with respect to preference shock ζ_d is even stronger than the one with respect to technology shocks z, which requires that hiring costs $\chi z f_l + \kappa_v$ are proportional to z.

Comparative Statics

To get more insight how changes in preferences and technology work through the model, it is helpful to undertake the comparative statics analysis before proceeding to the quantitative analysis of the business cycle properties of the model. As argued in Mortensen and Nagypal (2007) and Pissarides (2009), since measured labor productivity changes are rather persistent and labor market flows are large, the approximation of dynamics of the DMP model by its steady state elasticities is reasonably accurate.

Changes in Technology

Since actual technology z is not directly observable, and only measured average labor productivity y is observed, the relevant elasticity is ϵ^θ_y rather than ϵ^θ_z. The following lemma and proposition thus first restate the equilibrium goods and labor market conditions (1.3.1) and (1.3.2) in terms of measured average labor productivity. Then, they establish the relationship between the steady state elasticities of labor market tightness with respect to measured labor productivity in the labor search model $\epsilon_y^{\theta^{LS}}$, and in the goods and labor search model $\epsilon_y^{\theta^{GLS}}$.

Lemma 1. *Under Assumption 2 in the steady state (1.3.1) and (1.3.2) can be rewritten as*

$$0 = (1 - \epsilon_Q^{\psi^d})U_C C + U_D D \tag{1.3.6}$$

$$0 = \Big((1-\mu)f_L - \Big(\mu\theta + \frac{1-\beta(1-\delta)}{\beta\pi^v}\Big)(\chi f_L + \bar{\kappa}_V)\Big)\frac{N}{f-\bar{\kappa}}y + (1-\mu)\frac{U_N}{\epsilon_Q^{\psi^d} U_C} \tag{1.3.7}$$

and the real wage (1.3.5) as

$$\frac{w}{p} = \mu(f_L + \theta(\chi f_L + \bar{\kappa}_V))\frac{N}{f-\bar{\kappa}}y - (1-\mu)\frac{U_N}{\epsilon_Q^{\psi^d} U_C} \tag{1.3.8}$$

Assuming $\mu^{GLS} = \mu^{LS}$ and provided that all denominators are non-zero

$$\frac{1}{\epsilon_y^{\theta^{GLS}}} = \frac{1}{\epsilon_y^{\theta^{LS}}} + \frac{1}{1+\epsilon_C^{MRS_{CN}}}\left(\frac{1-\sigma}{\sigma}\epsilon_D^{m^G}\epsilon_z^{QX} + \epsilon_D^{MRS_{CN}}\epsilon_z^{Q}\right)\frac{1}{\epsilon_z^{\theta^{GLS}}} \tag{1.3.9}$$

Proposition 4 (Amplification - shocks to technology)**.** *Suppose that $\mu^{GLS} = \mu^{LS}$. Under Assumptions 1A and 2 $\epsilon_y^{\theta^{GLS}} > \epsilon_y^{\theta^{LS}}$ as long as m^G has elasticity of substitution $\sigma > 1$. Under Assumptions 1B and 2 $\epsilon_y^{\theta^{GLS}} > \epsilon_y^{\theta^{LS}}$ as long as u has relative risk aversion coefficient $\eta \leq 1$ and $\sigma \geq 1$; or alternatively $\eta \leq 1$ and $\sigma < 1$ and $\epsilon_D^{g_D}$ is sufficiently small.*

As (1.3.9) shows, in addition to the channel that works through higher steady state value of non-market activity and thus lower bargaining power, goods market frictions introduce two other channels which can result in amplification of effects that changes in technology have on labor market tightness. These two additional effects work through changes in the value of non-market activity over the business cycle, as a result of goods market search frictions.

First, there is an effect similar to the one that creates a hump shaped profile of consumption in a lifecycle model with preferences that are not separable between consumption and leisure. If preferences are not additively separable and changes in search effort affect marginal rate of substitution between consumption and hours worked, an increase in z causes an additional effect captured in (1.3.9) by the term $\epsilon_D^{MRS_{CN}} \epsilon_z^Q$. In particular, with GHH preferences from Assumption 1B if $|\epsilon_C^{U_C}| < 1$ consumption and search effort both increase in response to an increase in z. Because of the non-separability in utility, with increased search effort the marginal utility of consumption decreases less compared to the standard labor search model; this in turn means a smaller increase in the value of non-market activity, smaller upward pressure on wage in (1.3.8), and larger incentives for firms to hire new workers.

Second, there is an effect related to the amount of search effort needed to acquire one unit of consumption good changes over the business cycle. If the elasticity of substitution of the goods market matching function is different from one, changes in D and X affect how severe goods markets frictions are, in the sense that they change the elasticity $\epsilon_Q^{\psi^d}$. An increase in z then results in an additional effect captured in (1.3.9) by the term $\frac{1-\sigma}{\sigma} \epsilon_D^{m^G} \epsilon_z^{QX}$. To see how this effect works, consider the case where output supplied X and search effort D both increase in response to an increase in z. These two have in general opposing effects on $\epsilon_Q^{\psi^d}$; however, for additively separable preferences from Assumption 1A it holds that $\epsilon_z^{QX} \geq 0$ with equality if $\epsilon_D^{g_D} = 0$ and $\epsilon_C^{U_C} = 0$, and for GHH preferences from Assumption 1B similarly $\epsilon_z^{QX} \geq 0$ with equality if $\epsilon_D^{g_D} = 0$. As a result if $\sigma > 1$ and productivity z increases, goods markets frictions become less severe, that is $\epsilon_Q^{\psi^d}$ increases; this implies a smaller increase in the value of non-market activity, smaller upward pressure on wage in (1.3.8), and larger incentives for firms to hire new workers.

Changes in Preferences

Consider next the effects of a change in preference parameter ζ_d. Lower disutility from search results in higher search effort, which increases output and measured productivity even if technology z and employment would remain constant. This induced change in measured labor productivity y will however affect firms' incentives to hire new workers, and thus also labor market tightness θ. Similar to the case with changes in technology z discussed above, the relevant elasticity is $\epsilon_y^{\theta^{GLS}}$ rather than $\epsilon_{\zeta_d}^{\theta^{GLS}}$. The relationship between steady state elasticity $\epsilon_y^{\theta^{LS}}$ in response to a change in technology z in the labor search model, and the steady state elasticity $\epsilon_y^{\theta^{GLS}}$ in response to a change in preferences ζ_d in the model with goods and labor search can be summarized as follows.

Proposition 5 (Amplification - preference shocks)**.** *Suppose that $\mu^{GLS} = \mu^{LS}$. Under Assumptions 1A and 2 $\epsilon_y^{\theta^{GLS}} \gtreqless \epsilon_y^{\theta^{LS}}$ when m^G has elasticity of substitution $\sigma \lesseqgtr 1$.*

Consider a decrease in disutility ζ_d. When search effort by consumers and supply of goods and services by firms are complements in the matching function m^G, higher search effort results in larger incentives for firms to hire more workers in order to increase production, which is now more likely to be sold. Moreover, lower disutility from search per unit of good purchased $\frac{U_D}{\psi^d}$ provides additional incentives to hire more workers, since it creates a downward pressure on wages. The overall effect of the change in measured productivity on labor market tightness is larger than in the model with labor search only. When search effort and supply of goods and services are substitutes in the matching function m^G, incentives for firms to hire more workers in order to increase production are smaller, increase in search effort in much larger, and the result is actually a decrease in ψ^d and an increase in disutility from search per unit of good purchased $\frac{U_D}{\psi^d}$. The overall effect a same change in measured productivity on labor market tightness is consequently smaller than in the model with labor search only.

Worker's Outside Option and Bargaining Power

As already briefly mentioned above, goods market frictions provide some justification for the calibration in Hagedorn and Manovskii (2008), in particular for the choice of a high value of outside option of the worker and a low worker's bargaining power. To see this, consider an extension of the model where in addition to having more leisure, unemployed workers are engaged in home production and also receive unemployment benefit pb financed by a lump sum tax. Consumption c is then a composite good given by $c = g(c_m, c_n)$, where c_m is the amount of market goods and services and $c_n = h(1-n)$ is the amount of home produced goods and services. The wage in this economy is a small modification of (1.3.5)

$$\frac{w}{p} = \mu\psi^x(zf_L + \theta(\chi z f_L + \kappa_V)) + (1-\mu)\Big(b + \frac{U_C g_{c_n} h_{1-n}}{U_C g_{c_m} + \frac{U_D}{\psi^d}} - \frac{U_N}{U_C + \frac{U_D}{\psi^d}}\Big)$$

and the goods and labor market conditions (1.3.1) and (1.3.2) in the steady state become

$$0 = (1-\epsilon_Q^{\psi^d})\psi^d g_{c_m} U_C + U_D$$
$$0 = \Big((1-\mu)zf_L - \Big(\mu\theta + \frac{1-\beta(1-\delta)}{\beta\pi^v}\Big)(\chi z f_L + \kappa_v)\Big)\psi^x - (1-\mu)\Big(b + \frac{g_{c_n} h_{(1-n)}}{\epsilon_Q^{\psi^d} g_{c_m}} - \frac{U_N}{\epsilon_Q^{\psi^d} U_C}\Big)$$

The steady state condition for labor market implies that the outside option of the worker gets larger, when goods market search frictions become more severe and $\epsilon_Q^{\psi^d}$ becomes smaller. Arguably, given that the productivity of workers in home production and the process through which market and nonmarket goods are combined into the composite consumption good remain unchanged, the bargaining power of the worker μ thus has to be lower, if the same economy is viewed through the lens of the model with goods and labor search, rather than the standard labor search model. This provides yet another channel, in addition to those in Proposition 4, through which goods market frictions amplify effects of changes in productivity: As shown in Hagedorn and Manovskii (2008), increasing the value of non-market activity and at the same time decreasing worker's bargaining power to maintain the same steady state wage leads to wages which are less procyclical, and thus vacancies and unemployment which respond more to changes in productivity.

1.4 Quantitative Analysis

I consider the case with additively separable preferences

$$\mathrm{u}(c,d,e,\zeta) = \zeta_c \frac{c^{1-\eta}}{1-\eta} - \zeta_d \frac{d^{1+\varphi}}{1+\varphi} - \zeta_n e$$

Firms operate production technology given by $zf(l) = zl^\lambda$; labor and goods market matching functions are $m^L(U,V) = B(\gamma U^{\frac{\nu-1}{\nu}} + (1-\gamma)V^{\frac{\nu-1}{\nu}})^{\frac{\nu}{\nu-1}}$ and $m^G(D,TX) = A\big(\alpha D^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(TX)^{\frac{\sigma-1}{\sigma}}\big)^{\frac{\sigma}{\sigma-1}}$. To calculate steady state elasticities from Section 1.3 values for following parameters have to be set: $\beta,\eta,\varphi,\lambda,\mu,\delta,\gamma,\nu,B,\alpha,\sigma,A,\bar{\zeta}_c,\bar{\zeta}_d,\bar{\zeta}_n,\bar{z}$. For the dynamic simulation of the model in addition processes for $z,\zeta_c,\zeta_d,\zeta_n$ also have to be specified. In this section I first describe targets chosen to be matched in the U.S. data to calibrate the above parameters, and then describe the Bayesian estimation procedure used to estimate parameters of the processes for $z,\zeta_c,\zeta_d,\zeta_n$.

1.4.1 Calibration

Table 1.1 summarizes the targets and the parameter vales for the benchmark calibration. One period of the model is one fourth of a quarter, so roughly a week, and parameter β is chosen to obtain steady state annual interest rate of 5%. I set $\bar{z}$ to normalize the level of realized output $Y = 1$ and consider the case with constant returns to scale so that $\lambda = 1$. For labor market matching function parameters, I follow Shimer (2010) by setting $\nu = 1$, $\gamma = 0.5$ which implies a symmetric Cobb-Douglas matching function. As in Shimer (2005) I set the value of unemployment benefits b to 0.4 of average labor productivity in the steady state. The case with non-zero unemployment benefits allows a more precise calibration of the outside option of the worker, and implies that even in the case with logarithmic preferences and goods market matching function with unit elasticity of substitution, both technology and preference shocks have an effect on unemployment. Next, as in Hagedorn and Manovskii (2008), I use weekly job finding rate $\pi^u = 0.139$ and separation rate $\delta = 0.0081$; these values imply a steady state unemployment rate $U = 0.055$. Silva and Toledo (2009) and Hagedorn and Manovskii (2008) provide estimates for average costs associated with recruiting, screening and interviewing needed to hire a new worker: the 1982 Employment Opportunity Pilot Project survey, the 1992 Small Business Administration survey, and the findings in Barron, Berger, and Black (1997) suggests that these costs are about 4.5% of new worker's quarterly wages paid. Since a vacancy in the model attracts π^v workers to get one worker $\frac{1}{\pi^v}$ vacancies are needed. To match the above estimated hiring costs, if w is the weekly wage in the model, the total costs of a hire are $\frac{1}{\pi^v} w = 0.045 \times 12 \times w$. Thus for a weekly model I target $\pi^v = \frac{1}{12}\frac{1}{0.045} = 1.8519$. Given the job finding and recruitment rates targeted, since $\frac{\pi^u}{\pi^v} = \theta$ and $\pi^u = B\theta^{1-\gamma}$ the matching efficiency parameter is $B = (\pi^u)^\gamma (\pi^v)^{1-\gamma} = 0.507$.

For the benchmark I set the preference parameters so that $\varphi = 0$ and η targets intertemporal elasticity of substitution equal to 1. I set $\bar{\zeta}_c = 1$ and calibrate $\bar{\zeta}_d$ to normalize the steady state goods market tightness to $Q = 1$. The calibration of labor market matching function parameters above is based on direct empirical estimates (see Pissarides & Petrongolo, 2001), similar studies are unfortunately not available for the goods market matching function. Bai et al. (2012) and Bai and Ríos-Rull (2013) assume a Cobb-Douglas matching functions with elasticity with respect to demand $\alpha = 0.09$

Table 1.1: Calibration

	value	target or source	value
β	0.999	annual interest rate	5%
λ	1.00		
γ	0.50	Shimer (2010)	
μ	0.50	Hosios condition	
δ	0.0081	quarterly employment exit prob.	0.100
B	0.507	quarterly recruitment cost	$0.045w$
α	0.20		
A	0.848	capacity utilization rate	0.81
$\bar{z}$	1.313	output	1
ζ_n	0.412	unemployment rate	0.055
ζ_d	0.236	goods market tightness	1.00

and $\alpha = 0.25$ respectively, Petrosky-Nadeau and Wasmer (2011) use symmetric Cobb-Douglas matching function with elasticity 0.5. In the benchmark calibration of the goods market matching function I thus set $\sigma = 1$, $\alpha = 0.2$ and calibrate A to obtain steady state fraction of goods purchased $\frac{C}{X} = 0.81$. To get an idea how much the elasticity of substitution in the goods market matching function matters for the quantitative results, I then also consider alternative cases with $\sigma = 0.5$ and $\sigma = 2$.

Finally, to set $\bar{\zeta}_n$ notice that for a given bargaining power μ, value of home production and leisure ζ_n affects wage and through that profits of the firms, hiring, labor tightness θ, and also U. For the model without goods market friction I thus proceed as Shimer (2010), set $\mu = \gamma$ and calibrate $\bar{\zeta}_n$ to match the above mentioned target unemployment rate $U = 0.055$. In the model with goods market search parameters μ and ζ_n can then be set in two alternative ways. In the first μ is kept unchanged and value of home production and leisure ζ_n is recalibrated to maintain the same steady state labor market tightness θ. In the second one ζ_n is kept unchanged and μ is recalibrated. I use the first approach in order to quantify the amplification effect of goods market frictions beyond the effect implied by a lower bargaining power as discussed in Section 3.2.2 above.

1.4.2 Steady State Elasticities

Table 1.2 compares the steady state elasticity of vacancy-unemployment ratio with respect to measured average labor productivity ϵ_y^θ for the goods and labor search model in this paper, elasticities from existing labor search models in related papers, and also the empirical counterpart of this elasticity based on the data for U.S economy.

As shown in the top panel of Table 1.2, the standard deviation of log of the vacancy-unemployment ratio in U.S. for the 1951 to 2003 period is 19.1 times larger than the standard deviation of log average labor productivity. In contrast, as shown in the first line of the second panel the steady state elasticity ϵ_y^θ in Shimer (2005) is only 1.71. Subsequent papers by Hall (2005), Hall and Milgrom (2008) obtain elasticity in their models even larger than the target in the data, by modifying the wage determination mechanism to get less procyclical wages. Hagedorn and Manovskii (2008) maintain Nash bargaining and are able to generate the right amount of fluctuations through different calibration, by setting workers' bargaining power to $\mu = 0.052$ and the value of unemployment to $b = 0.955$. This large value of unemployment however implies a semielasticity of unemployment to changes in unemployment benefits replacement ratio ϵ_b^U which is seven times larger than what is empirically observed in U.S. data (Costain & Reiter, 2008).

Pissarides (2009) and Mortensen and Nagypal (2007) point out that $\frac{\sigma_\theta}{\sigma_y} corr(\theta, y)$ is a more appropriate target then a simple ratio $\frac{\sigma_\theta}{\sigma_y}$, to evaluate any model where productivity shocks are the only source driving of fluctuations. Arguably, other shocks, to preferences, matching efficiency, separation rate, bargaining power or interest rates can to some extent be the reason behind the fluctuations in vacancy-unemployment ratio observed in data. The choice of $\frac{\sigma_\theta}{\sigma_y} corr(\theta, y)$ as a target is then justified, because this would be the coefficient obtained by running a regression of log of the vacancy-unemployment ratio on log average labor productivity. This yields 7.56 as a target against which Pissarides (2009) and Mortensen and Nagypal (2007) compare the steady state elasticity ϵ_y^θ in their versions of the labor search model which feature labor turnover costs as an additional element. Both papers show that the amount of fixed training costs needed to achieve the target value for the elasticity is quite plausible, in the range of 20% to 40% of the quarterly output of the match. Silva and Toledo (2013) however point out that the crucial detail that matters is the fraction of the labor turnover costs that are

Table 1.2: Comparison of models based on steady state elasticity ϵ_y^θ

US data (1951:2003 period, from Shimer (2005))	
$\frac{\sigma_\theta}{\sigma_y}$	19.10
$\frac{\sigma_\theta}{\sigma_y} corr(\hat{y}, \theta)$	7.56

Labor search models	ϵ_y^θ
Shimer (2005)	1.71
Hall (2005)	81.70
Hall and Milgrom (2008)	42.35
Hagedorn and Manovskii (2008)	23.72
Mortensen and Nagypal (2007)	7.56
Pissarides (2009)	7.25
Silva and Toledo (2013)	4.17

Benchmark labor search model, $\alpha = 0$	ϵ_y^θ
$\eta = 1$, $b = 0.4$	3.69

Goods and labor search model, $\alpha = 0.2$	ϵ_y^θ
$\eta = 1$, $b = 0.4$, $\sigma = 2$	z: 5.03 and ζ_d: 1.10
$\eta = 1$, $b = 0.4$, $\sigma = 0.5$	z: 2.82 and ζ_d: 9.05

sunk at the point when the match is created. In addition, they show that increasing the labor turnover costs has a similar effect on the response of unemployment to changes in unemployment benefits as an increase in the value of unemployment in Hagedorn and Manovskii (2008). Using the available empirical evidence on training costs to discipline the calibration, in addition to restricting the semielasticity ϵ_b^U empirically observed in U.S. data, they find no amplification mechanism generated by fixed labor turnover costs. Their value of elasticity $\epsilon_y^\theta = 4.17$ in the model with labor turnover costs is essentially the same as $\epsilon_y^\theta = 4.18$ in the model without these costs, and is also very close to no labor turnover costs benchmark from Mortensen and Nagypal (2007) where $\epsilon_y^\theta = 3.89$ and Pissarides (2009) where $\epsilon_y^\theta = 3.67$.

Calibration of the benchmark model with labor search in this paper results in elasticity of similar magnitude since $\epsilon_y^{\theta^{LS}} = 3.69$. In comparison, in a model with goods and labor search, $\epsilon_y^{\theta^{GLS}}$ is about 40% larger when the driving force is a productivity shock and goods market matching function has elasticity of substitution $\sigma = 2$, and about 150% larger when the driving force is a preference shock and goods market matching function has elasticity of substitution $\sigma = 0.5$. This amplification is in line with theoretical results in Proposition 4 and Proposition 5.

1.4.3 Estimation

To specify the parameters for shock processes $\zeta_c, \zeta_d, \zeta_n, z$, I first consider the model with only one shock at a time; and the process considered is $\log x' = (1-\rho_x)\log \bar{x} + \rho_x \log x + e'_x$ for each shock $x \in \{z, \zeta_c, \zeta_d, \zeta_n\}$. To obtain the autocorrelation coefficients ρ_x and variance of innovations σ_x^2, I estimate a log-linearized weekly model using Bayesian methods, to match quarterly time series for average labor productivity.[2] The labor productivity measure used for estimation is 1951Q1-2010Q4 output per worker in nonfarm business sector. Quarterly labor productivity y_t is calculated as quarterly output Y_t divided by the quarter's employment N_t. Quarterly output is the sum of weekly output, and quarterly employment is given by the average employment in the three months of the quarter.

Table 1.3 and Table 1.4 show the choice of prior distributions, the estimated posterior mode obtained by maximizing the log of the posterior distribution with respect to the parameters, the approximate standard error based on the corresponding Hessian, and also the mean, mode, 10 and 90 percentile of the posterior distribution of the parameters obtained through the random walk Metropolis-Hastings sampling algorithm with four chains and 100000 draws. The estimated standard deviations for ζ_c and ζ_n shocks in the model without goods market search are very large, much larger than in the model with both search frictions. This is to be expected since the only channel through which they can generate movements in measured labor productivity y is through their effect on θ and productive workforce $L = N - \theta(1 - N)$. Thus recruitment needs to vary a lot to match the measured labor productivity, which requires big shocks to preferences.

[2] See An and Schorfheide (2007), Del Negro and Schorfheide (2008) and Lubik (2009) for details regarding Bayesian estimation.

Table 1.3: Parameter estimates for labor search model

		Prior		Posterior		
		mean	st.dev.	mode	mean	90 % HPD interval
ρ_c	Beta	0.900	0.05	0.9965	0.9963	[0.9945,0.9981]
σ_c	Inverse Gamma	0.010	20.00	0.2552	0.2576	[0.2367,0.2782]
	Log data density 792.27					
ρ_n	Beta	0.900	0.05	0.9964	0.9963	[0.9945,0.9981]
σ_e	Inverse Gamma	0.010	20.00	0.2531	0.2553	[0.2346,0.2749]
	Log data density 792.28					
ρ_z	Beta	0.900	0.05	0.9956	0.9954	[0.9934,0.9976]
σ_z	Inverse Gamma	0.010	20.00	0.0035	0.0035	[0.0033,0.0038]
	Log data density 796.99					

Table 1.4: Parameter estimates for goods and labor search model

		Prior		Posterior		
		mean	st.dev.	mode	mean	90 % HPD interval
ρ_c	Beta	0.900	0.05	0.9962	0.9960	[0.9941,0.9979]
σ_c	Inverse Gamma	0.010	20.00	0.0198	0.0200	[0.0185,0.0215]
	Log data density 797.27					
ρ_d	Beta	0.900	0.05	0.9957	0.9956	[0.9935,0.9976]
σ_d	Inverse Gamma	0.010	20.00	0.0175	0.0176	[0.0163,0.0189]
	Log data density 796.47					
ρ_n	Beta	0.900	0.05	0.9930	0.9928	[0.9899,0.9959]
σ_n	Inverse Gamma	0.010	20.00	0.1336	0.1349	[0.1237,0.1462]
	Log data density 777.72					
ρ_z	Beta	0.900	0.05	0.9957	0.9955	[0.9935,0.9976]
σ_z	Inverse Gamma	0.010	20.00	0.0044	0.0044	[0.0041,0.0048]
	Log data density 797.71					

1.4.4 Business Cycle Moments

Unitary Elasticity of Substitution

Table 1.5 and Table 1.6 show the results of the simulation of models with and without goods market friction, with parameters of shocks set at their posterior means. Comparing panels (A) and (B) in Table 1.5 we can see that the large shocks to ζ_c required to generate the observed movements in labor productivity cause fluctuation in labor market tightness and recruitment which are 20 times higher than in the data. Moreover, the correlations of all variables with measured labor productivity y have wrong signs - if z is constant, for measured labor productivity y to increase, productive labor $L = N - \theta(1 - N)$ has to increase relative to overall labor N, and thus θ has to fall. Shocks to disutility from work ζ_n suffer from the same problem. Thus without goods market frictions technology shocks are the only plausible source of business cycle fluctuations in this model.

Table 1.5: U.S. data and labor search model

	(A) U.S. data				(B) Consumption utility shock ζ_c			
	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.
y	0.013	1.00	0.67	0.76	0.013	1.00	-0.89	0.75
θ	0.266	0.34	0.88	0.91	5.060	-0.98	0.89	0.77
V	0.141	0.42	0.89	0.91	3.137	-0.92	0.71	0.60
U	0.131	-0.24	-0.83	0.89	2.282	0.91	-1.00	0.83
	(C) Labor disutility shock ζ_n				(D) Productivity shock z			
	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.
y	0.013	1.00	-0.89	0.75	0.014	1.00	1.00	0.78
θ	5.011	-0.98	0.89	0.77	0.051	0.99	0.99	0.78
V	3.101	-0.92	0.71	0.60	0.031	0.93	0.91	0.63
U	2.262	0.91	-1.00	0.83	0.023	-0.94	-0.95	0.83

Once goods market search is introduced into the model the situation changes considerably. Panels (A) and (D) of Table 1.6 document the observational equivalence of technology shocks z and preference shocks ζ_d from Proposition 2: shocks to disutility from search in goods market generate the same fluctuations as technology shocks. Comparing panels (D) for the two economies, with and without goods market search reveals that the volatility of labor market tightness is basically the same in both economies.

Table 1.6: Summary statistics, goods and labor search model, $\sigma = 1$

	(A) Shopping disutility shock ζ_d				(B) Consumption utility shock ζ_c			
	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.
y	0.014	1.00	1.00	0.78	0.013	1.00	0.98	0.76
θ	0.050	0.99	0.99	0.78	0.465	0.99	0.98	0.78
V	0.031	0.93	0.91	0.63	0.284	0.94	0.87	0.63
U	0.023	-0.94	-0.95	0.83	0.212	-0.92	-0.98	0.83
	(C) Labor disutility shock ζ_n				(D) Productivity shock z			
	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.
y	0.018	1.00	-0.99	0.82	0.014	1.00	1.00	0.78
θ	2.492	-0.95	0.89	0.77	0.050	0.99	0.99	0.78
V	1.542	-0.80	0.71	0.60	0.031	0.93	0.91	0.63
U	1.127	0.99	-1.00	0.83	0.023	-0.94	-0.95	0.83

This is also in line with theoretical analysis in the previous ch1:sec: Proposition 4 proved that for steady state elasticity ϵ_y^θ there is no amplification in the case with additively separable utility function and goods market matching function with unitary elasticity of substitution. For shocks to marginal utility of consumption ζ_c, the size of the shocks necessary to generate the observed movements in labor productivity falls once the goods market search is introduced. Moreover, correlations of all variables with measured labor productivity y have now correct signs, and fluctuations of labor market tightness and recruitment are closer to those in data.

Non-Unitary Elasticity of Substitution

Table 1.7 and Table 1.8 present the moments for the goods and labor search model with elasticity of substitution between D and X in the goods matching function of 0.5 and 2. They confirm the results from Proposition 4, which were already suggested by steady state elasticities in Table 1.2. In the case with high substitutability and technology shocks, the observed fluctuations in vacancy-unemployment ratio are about 30% larger, compared to the model with labor search only. In the case with low substitutability, preference shocks to disutility from search for goods result in observed fluctuations in vacancy-unemployment ratio that are about 130% larger.

Table 1.7: Summary statistics, goods and labor search model, $\sigma = 0.5$

	(A) Shopping disutility shock ζ_d				(B) Consumption utility shock ζ_c			
	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.
y	0.014	1.00	1.00	0.78	0.013	1.00	0.97	0.76
θ	0.121	0.99	0.99	0.78	0.782	0.99	0.96	0.78
V	0.074	0.93	0.90	0.63	0.479	0.94	0.84	0.62
U	0.055	-0.93	-0.96	0.83	0.356	-0.91	-0.99	0.83
	(C) Labor disutility shock ζ_n				(D) Productivity shock z			
	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.
y	0.017	1.00	-0.98	0.82	0.014	1.00	1.00	0.78
θ	3.051	-0.96	0.89	0.77	0.039	0.99	0.99	0.78
V	1.888	-0.83	0.71	0.60	0.024	0.92	0.92	0.63
U	1.379	0.99	-1.00	0.83	0.018	-0.94	-0.95	0.83

Table 1.8: Summary statistics, goods and labor search model, $\sigma = 2$

	(A) Shopping disutility shock ζ_d				(B) Consumption utility shock ζ_c			
	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.
y	0.014	1.00	1.00	0.78	0.013	1.00	0.99	0.76
θ	0.015	0.99	0.99	0.78	0.272	0.99	0.98	0.78
V	0.009	0.92	0.92	0.63	0.167	0.94	0.89	0.63
U	0.007	-0.94	-0.94	0.83	0.124	-0.92	-0.96	0.83
	(C) Labor disutility shock ζ_n				(D) Productivity shock z			
	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.	st.dev.	corr($\cdot$,y)	corr($\cdot$,Y)	autocorr.
y	0.018	1.00	-0.99	0.83	0.014	1.00	1.00	0.78
θ	1.869	-0.93	0.89	0.77	0.067	0.99	0.99	0.78
V	1.154	-0.78	0.71	0.60	0.041	0.93	0.91	0.63
U	0.846	1.00	-1.00	0.83	0.030	-0.93	-0.95	0.83

1.4.5 Impulse Response Analysis

Weekly impulse response functions to shocks that generate a one percent increase in measured labor productivity are shown in Figure 1.1. As expected given the results so far, the response to technology shock z and preference shock ζ_d are virtually identical for the case with unit elasticity of substitution. Only the behavior of goods market tightness is different. The other two cases imply either a stronger response of unemployment to technology shocks (if the elasticity of substitution between X and D is high) or to preference shocks (if the elasticity of substitution between X and D is low).

To examine further the dynamics of the labor market variables in the model, I next look at the impulse response functions for the model generated quarterly data, and compare them to their empirical counterparts. To that end, I first use quarterly U.S. data on labor productivity, vacancies, unemployment, and employment to estimate a reduced form VAR $\tilde{\mathbf{x}}_t = \sum_{i=1}^{4} \mathbf{A}_i \tilde{\mathbf{x}}_{t-i} + \varepsilon_t$ with $\tilde{\mathbf{x}}_t = (\tilde{y}_t, \tilde{\theta}_t, \tilde{N}_t)'$, where $\tilde{y}_t, \tilde{\theta}_t, \tilde{N}_t$ are the log transformed average labor productivity, vacancy-unemployment ratio and employment, detrended using a third order time polynomial. I then obtain the empirical impulse response functions to a one-standard deviation shock to productivity, using the Cholesky decomposition to orthogonalize shocks with an identification scheme where the shock to productivity is first in the ordering. Afterwards, I run 1000 simulations of the model, each time aggregate the data into quarterly time series and estimate the same VAR on this artificial data. Figure 1.2 compares the resulting average impulse response functions with empirical counterparts. The top panel shows the case where goods market matching function has unitary elasticity of substitution, and the response of employment and labor market tightness to an increase in the measured productivity is the same in the model without goods market search and with goods market search. This is again in line with results from Proposition 4 and Proposition 5. With non-unitary elasticity of substitution, the model with goods markets frictions performs better than the model with labor search only in terms of amplification, but the problem with the lack of propagation is still present. The response of employment to an increase in measured labor productivity in the model is on impact similar to the response in data, but while in the data employment further increases in the following quarters and the peak occurs after five quarters from the initial shock, in the model this build up is much less pronounced and rather short lived, with peak already in the third quarter.

Figure 1.1: Impulse response function, weekly model

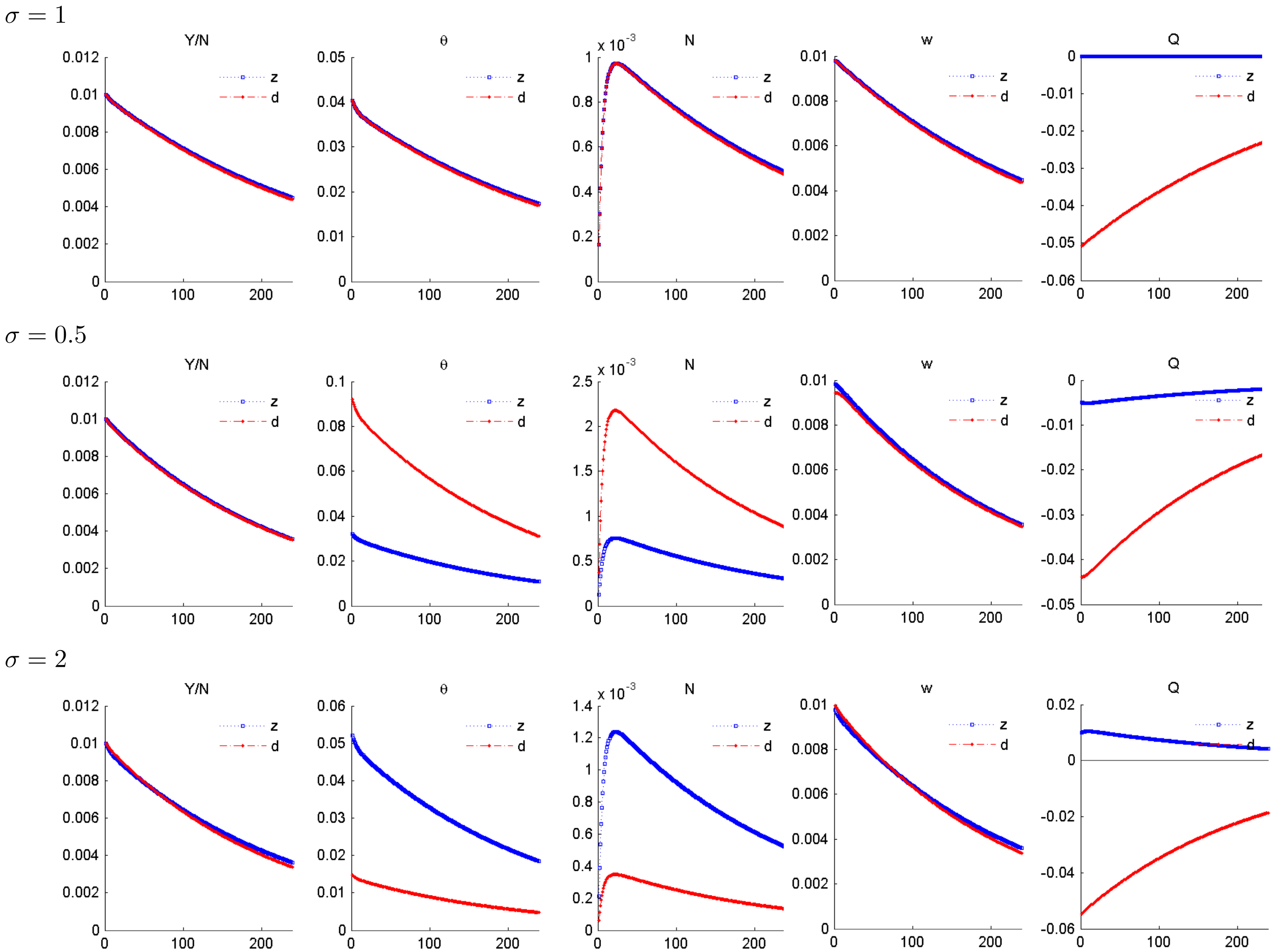
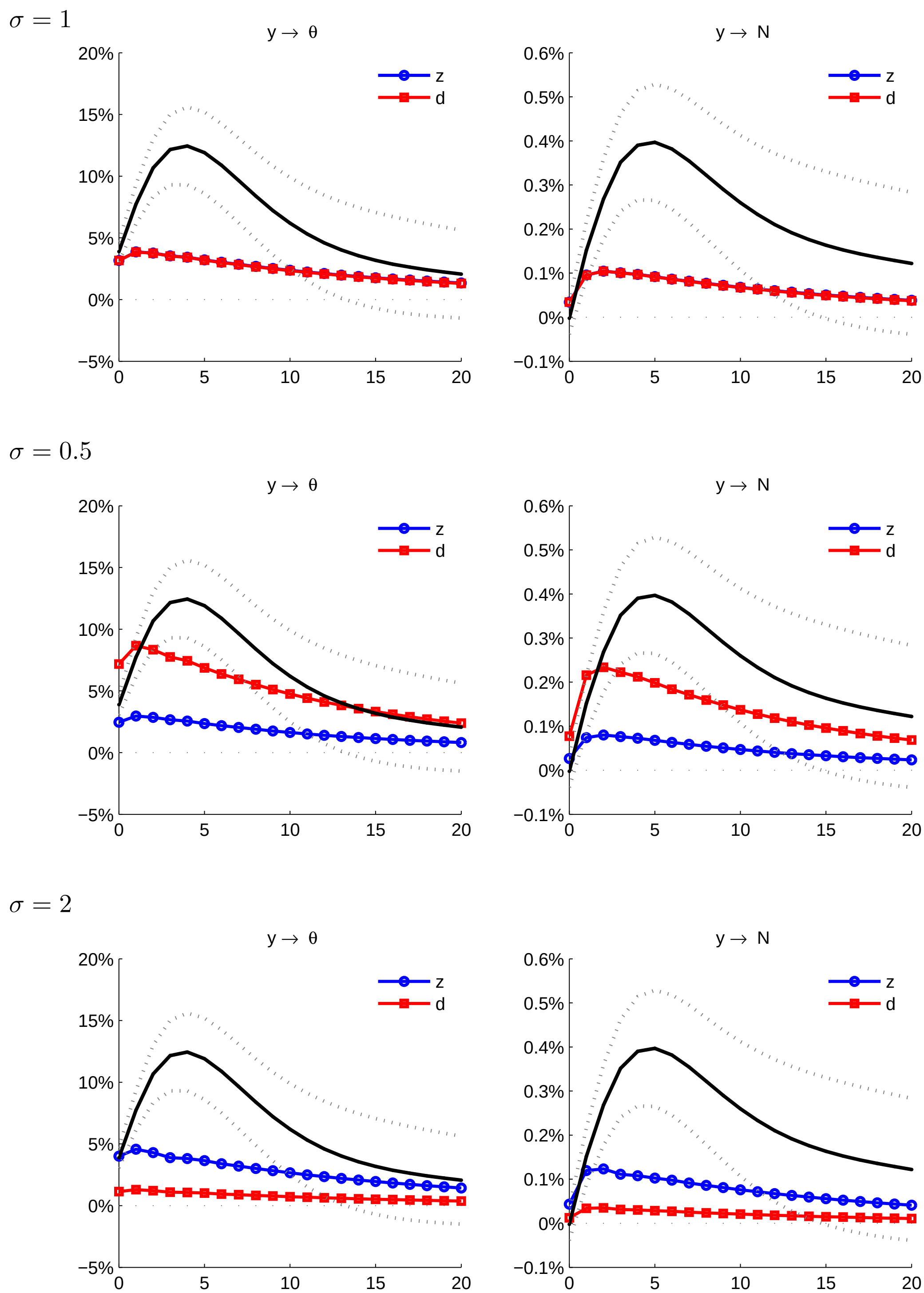


Figure 1.2: Impulse response function, quarterly data



1.5 Conclusion

This paper studies the fluctuations of unemployment in a framework that emphasizes the role of consumer demand in determining the output and employment. In particular, it examines the amplification of shocks in a Diamond-Mortensen-Pissarides model after goods market search-matching friction is introduced. When wages are determined by Nash bargaining, goods market frictions affect worker's bargaining position, provide rationale for high value of non-market activity, but also change its cyclical properties. This last effect arises since higher availability of goods in expansions makes frictions in the goods market less severe from consumer's perspective, thus increasing the value of additional earnings obtained when the worker accepts the job. In addition, in the framework analyzed in this paper supply side shocks to technology and demand side shocks to preferences can generate business cycle fluctuations that are observationally equivalent to an economist who would only consider time series for labor productivity, output, employment and wages.

I estimate the model first using data on U.S. average labor productivity, and show that a modest amount of goods market frictions increases the response of unemployment to technology shocks by one third when search effort and output supplied by firms are good substitutes in the goods market matching function. With low substitutability preference shocks result in response of unemployment which is about two and half times larger than the response to technology shocks in model with labor search only.

Chapter 2

Inventories and Unemployment

2.1 Introduction

As shown in Chapter 1, it is possible to obtain a significantly larger response of unemployment to changes in measured labor productivity by introducing goods market frictions into a labor search-matching model. But the previous chapter also shows that the magnitude of this amplification effect depends on the elasticity of substitution of the goods market matching function, and whether the change in measured labor productivity was caused by a technology or a preference shock. Moreover, using only data on labor productivity, output, employment or wages it is not possible to separately identify the technology and preference shocks, or the elasticity of substitution of the goods market matching function.

In this chapter I show that inventories, which naturally arise in an environment where search frictions prevent firms from selling all goods instantaneously, provide a way to determine the relative importance of technology and preference shocks. The two sources of fluctuations can be distinguished because technology and preference shocks have different implications for the response of inventory-sales ratio in the model. In particular, in a model without capital this ratio increases in response to a positive technology shock, but decreases in response to a preference shock. Intuitively, in the model with inventories a positive shock to technology results in a build up of inventories relative to sales if the demand does not increase; if on the other hand technology is unchanged and the shock decreases consumers' disutility from search, the result is a drop of inventories relative to sales. And because the size od the response of inventory-sales ratio depends on the elasticity of substitution in the goods market matching function σ, data on inventories and sales provide a source of identification for this parameter.

In the model estimated using data on labor productivity and inventory-sales ratio the response of vacancy-unemployment ratio is twice as large as in the model with labor search only. In addition, model can match the main facts on inventories - procyclical inventory investment, countercyclical inventories-sales ratio, and sales which are more volatile that production. The two empirical facts, that inventory-sales ratio is countercyclical, and that sales are more volatile than production turned out to pose quite a challenge in developing models that would be able to replicate them (see Ramey & West, 1999, Bils & Kahn, 2000 and Khan & Thomas, 2007b for further discussion on this issue). As shown here, a relatively simple model with goods market frictions and with demand and supply side shocks can actually match these facts quite well.

2.2 Inventories in the Model with Goods Market Search

I first extend the model from Chapter 1 by allowing firms to store goods that are not sold, in an attempt to sell them in the future. Thus let i be the stock of inventories at the beginning of period and i' the amount of goods carried over to the next period then

$$i' = (1 - \delta_i)(1 - \psi^x)x$$

where $\delta_i \in (0, 1)$ captures the loss of value due to obsoleteness, the fact that some goods will not be demanded at all in the future, and also the storage costs and the inability to store services. As a second, minor modification, production technology in the economy is subject to permanent and transitory shocks. The growth rate of the permanent component is

$$\gamma'_A = \frac{A'}{A}$$

and the aggregate supply of goods is

$$X = zAf(N - V) + I$$

To guarantee that the model is compatible with long run balanced growth, I assume that the permanent component of the technology also makes search effort by consumer in the goods market more productive; the amount of goods sold is thus $m^G(AD, TX)$.

The aggregate state now comprises of the shocks, measure of employed workers, and the stock of inventories at the beginning of period, $\mathbf{S} = (z, \zeta, A, N, I)$.

The problem of the firm is then a modification of (1.2.3)

$$\Omega(n, i; \mathbf{S}) = \max_{v,p,Q,x} \left\{ p\psi^x(Q, X(\mathbf{S}), A)x - w(\mathbf{S})n + \mathbb{E}[m(\mathbf{S}, \mathbf{S}')\Omega(n', i'; \mathbf{S}')] \right\}$$

subject to

$$x = zAf(n - v) + i$$

$$n' = (1 - \delta_n)n + \pi^v(\theta(\mathbf{S}))v$$

$$i' = (1 - \delta_i)(1 - \psi^x(Q, X(\mathbf{S}), A))x$$

$$W_d^*(\mathbf{S}) = U_d(\mathbf{S}) + (U_c(\mathbf{S}) - pM(\mathbf{S}))\psi^d(Q, X(\mathbf{S}), A)$$

$$\mathbf{S}' = G(\mathbf{S})$$

Presence of inventories in the model does not affect the household's problem, which remains essentially same as (1.2.2)

$$W(\mathbf{s}; \mathbf{S}) = \max_{c,d,a'} \left\{ U(c, d, n, \zeta) + \beta \mathbb{E} W(\mathbf{s}'; \mathbf{S}') \right\}$$

subject to

$$p(\mathbf{S})c + a' = (1 + R(\mathbf{S}))a + w(\mathbf{S})n + p(\mathbf{S})b(\mathbf{S})(1 - n) - t(\mathbf{S})$$

$$c = d\psi^d(Q(\mathbf{S}), X(\mathbf{S}), A)$$

$$n' = (1 - \delta_n)n + \pi^u(\theta(\mathbf{S}))(1 - n)$$

$$\mathbf{S}' = G(\mathbf{S})$$

The only difference in the households problem above comes from unemployment benefits, which as discussed in Section 1.4 are introduced to allow a more precise calibration of the outside option of a worker.

Following similar steps as in Appendix A and Appendix B one can obtain a system of equations for goods and labor markets tightnesses Q and θ

$$
\begin{aligned}
&-U_d = (1-\epsilon_Q^{\psi^d})\psi^d\left[U_c - (1-\delta_i)\beta\mathbb{E}\left[\Big(U_c' + \frac{U_d'}{(\psi^d)'}\Big)(\Omega_i^r)'\right]\right] \\
&\frac{1}{\pi^v} zAf_l\Omega_i^r\Big(U_c + \frac{U_d}{\psi^d}\Big) \\
&= \beta\mathbb{E}\left[\Big(1-\mu-\mu\theta' + \frac{1-\delta_n}{(\pi^v)'}\Big)z'A'f_l'(\Omega_i^r)'\Big(U_c' + \frac{U_d'}{(\psi^d)'}\Big) - (1-\mu)\left[\Big(U_c' + \frac{U_d'}{(\psi^d)'}\Big)b - U_n'\right]\right]
\end{aligned}
$$

where the real marginal value of inventories $\Omega_i^r = \frac{\Omega^i}{p}$ evolves according to

$$
\Omega_i^r = \psi^x + (1-\psi^x)(1-\delta_i)\beta\mathbb{E}\left[\frac{U_c' + \frac{U_d'}{(\psi^d)'}}{U_c + \frac{U_d}{\psi^d}}(\Omega_i^r)'\right]
$$

The stochastic difference equation for labor market tightness θ is a minor generalization of (1.3.2), because the marginal unit of output produced is now worth Ω_i^r instead of just ψ^x as in the model without inventories. The additional term in the goods market condition compared to (1.3.1) for the benchmark model captures the fact that increased effort lowers the stock of inventories and thus also the amount of goods that could be potentially purchased and consumed in the next period.

Finally, in the model with inventories the measured labor productivity is given by

$$
y = \frac{\psi^x X + I' - I}{N}
$$

Because of the stochastic trend A that increases productivity, some endogenous variable in the model economy are also growing. To stationarize the model, supply of goods by firms, consumption, output, and end of period stock of inventories need to be divided by the stochastic trend A. The equations that characterize the dynamics of model variables are then log-linearized; the resulting set of equations can be found in Appendix D.

2.3 Quantitative Analysis

2.3.1 Calibration

Calibration of the model with inventories largely follows Chapter 1. I again consider the case where workers have additively separable preferences

$$\mathrm{u}(c, d, e, \zeta) = \zeta_c \log c - \zeta_d d - \zeta_n e$$

firms operate production technology given by $zf(l) = zAl^{\lambda}$, and two matching functions, for the labor and goods markets are $m^L(U,V) = B(\gamma U^{\frac{\nu-1}{\nu}} + (1-\gamma)V^{\frac{\nu-1}{\nu}})^{\frac{\nu}{\nu-1}}$ and $m^G(AD, TX) = \left(\alpha AD^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(TX)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$.

A period in the model is a month and parameter β is chosen to obtain annual interest rate of 5%. I set $\bar{z}$ to normalize consumption $C = 1$ and consider the case with constant returns to scale so that $\lambda = 1$.

For labor market matching function parameters, I again set $\nu = 1$, $\gamma = 0.5$ which implies a symmetric Cobb-Douglas matching function. As in Shimer (2005) I set b to obtain unemployment benefits on balanced growth path equal to 0.4 of average labor productivity. The monthly finding rate is $\pi^u = 0.58$ and the separation rate $\delta_n = 0.034$; these values imply unemployment rate on balanced growth path $U = 0.055$. Following Silva and Toledo (2009) and Hagedorn and Manovskii (2008) I target the average costs associated with recruiting, screening and interviewing needed to hire a new worker as 4.5% of new worker's quarterly wages paid. Since a vacancy in the model attracts π^v workers to get one worker $\frac{1}{\pi^v}$ vacancies are needed. To match the above hiring costs, if w is the monthly wage in the model, the total costs of a hire are $\frac{1}{\pi^v}w = 0.045 \times 3 \times w$. Thus $\pi^v = \frac{1}{3}\frac{1}{0.045} = 7.4074$. Given the job finding and recruitment rates targeted, since $\frac{\pi^u}{\pi^v} = \theta$ and $\pi^u = B\theta^{1-\gamma}$ the matching efficiency parameter is $B = (\pi^u)^{\gamma}(\pi^v)^{1-\gamma} = 2.08$. I set $\mu = \gamma$ and calibrate $\bar{\zeta}_n$ to match the above mentioned target unemployment rate $U = 0.055$.

I calibrate $\bar{\zeta}_d$ to normalize the goods market tightness on balanced growth path to $Q = 1$, assume a symmetric matching functions with $\alpha = 0.5$. Parameter δ_i is chosen to reflect that for services sector depreciation is effectively 1, and for goods sector the quarterly depreciation is 0.15. As services constitute three quarters of the output of model economy, the implied overall monthly depreciation of inventories is 0.763.

2.3.2 Estimation

All three exogenous driving forces in the model are assumed to follow an AR(1) process, thus

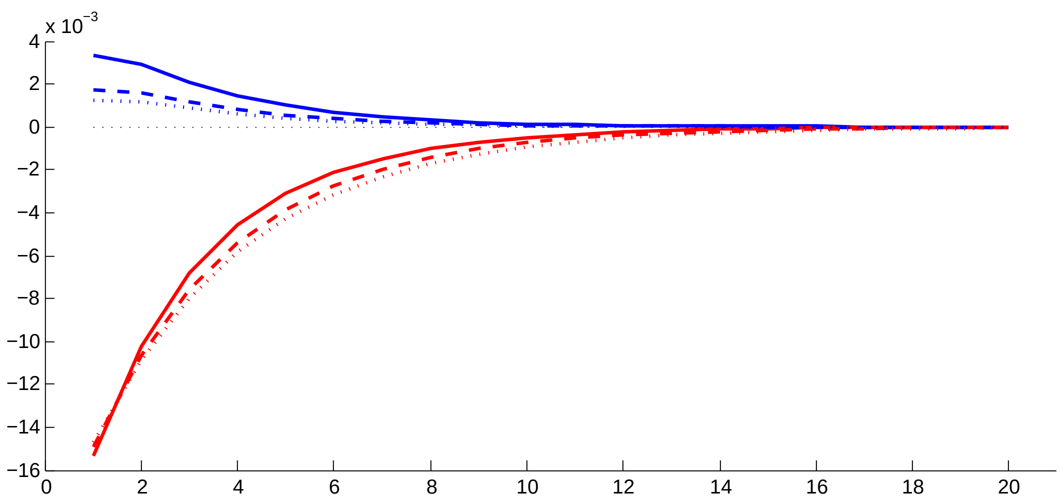
$$\log \xi' = (1-\rho_\xi)\log \bar{\xi} + \rho_\xi \log \xi + \epsilon'_\xi,$$

for $\xi \in \{\gamma_A, z, \zeta_d\}$ where $\epsilon'_\xi \sim N(0, \eta_\xi^2)$.

The two time series used in the Bayesian estimation are the quarterly growth rate of the average labor productivity $\gamma_y = \Delta \log \frac{Y}{N}$ and the ratio of inventories to sales $\iota = \frac{I}{C}$. Average labor productivity is seasonally adjusted real output in nonfarm business sector constructed by the BLS from NIPA divided by employment form the monthly Current Population Survey, the ratio of inventories to sales is constructed using data for real nonfarm inventories and real final sales of domestic business from BEA. The sample used for estimation spans the period 1951Q1-2010Q4.

As shown in Figure 2.1, the behavior of inventories to sales ratio is qualitatively different in response to technology and preference shocks. In addition, elasticity of substitution in the goods market matching function matters quantitatively for the magnitude of response. Inventories to sales ratio thus contains information that can be used to determine the contribution of the two shocks to the business cycle fluctuations, and to identify the elasticity of substitution in the goods market matching function.

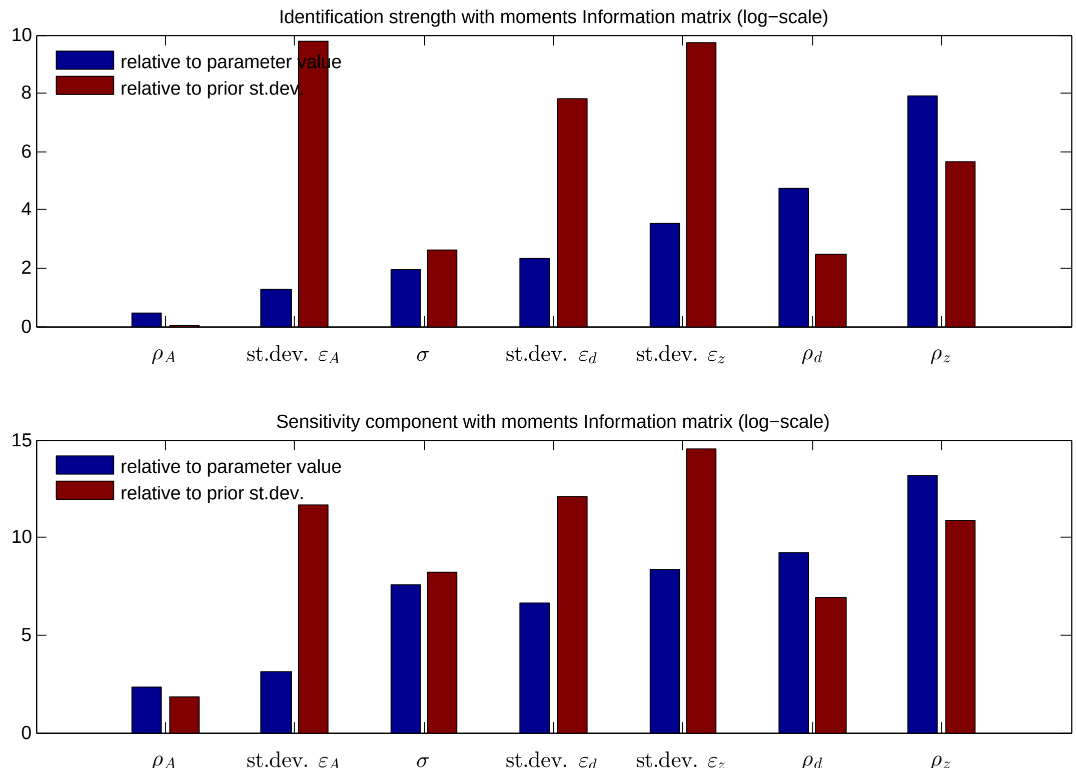
Figure 2.1: Impulse response functions of inventories to sales ratio for different shocks



Blue z, red ζ_d. Full lines $\sigma = 0.5$, dashed lines $\sigma = 1$, dotted lines $\sigma = 2$.

I estimate the parameters of the processes for γ_A, z and ζ_d and the elasticity of substitution in the goods market matching function σ. To verify that all parameters that are estimated are identified, I use the identification tests proposed in Iskrev (2010b).

Figure 2.2: Identification strength and sensitivity analysis



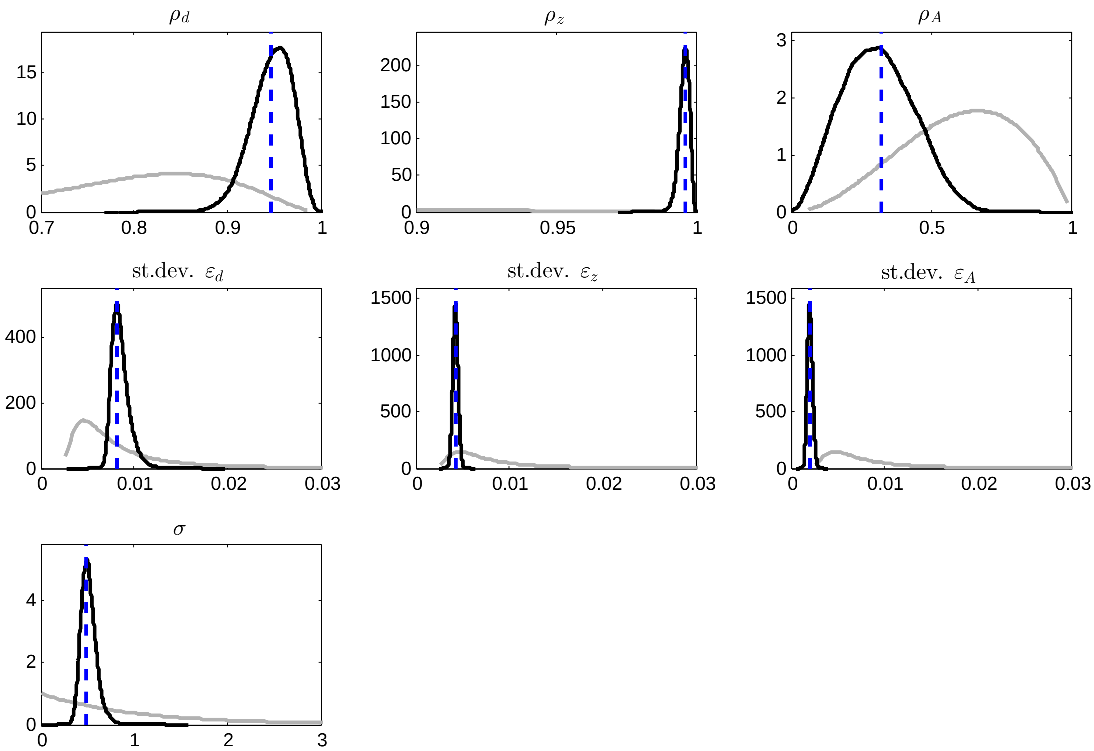
These test is based on the idea that the autocovariogram of the observables with respect to the vector of estimated parameters should have rank equal to the number of the estimated parameters. For posterior mean, I find that the Jacobian matrices J_2 and $J(q)$ employed in the tests both have full rank already with just one lag included, and so parameters are locally identified both in the model and in the data used for estimation. Another issue that can arise in estimation is that some parameters are identified only weakly due to either small sensitivity of moments in the data to that parameter, or due to the high collinearity among some column in Jacobian matrices J_2 and $J(q)$. It is thus useful to inspect identification strength measures from Iskrev (2010a), and the singular value decomposition of the Fisher information matrix as proposed by Andrle (2010). Figure 2.2 shows the identification strength measures and orders parameters according to the strength of their identification. All estimated parameters affect the behavior of the model, but there is some colinearity present that results in a somewhat weaker identification of parameters of the AR(1) process for the growth rate of the stochastic trend γ_A. This is confirmed by singular value decomposition pattern where the smallest singular value is associated with parameter ρ_A.

Posterior distributions for parameters are obtained by random walk Metropolis-Hastings Estimation algorithm with four chains and 500000 draws, with fist half of draws disregarded as burn-in. The results of estimation are shown in Table 2.1. All parameters are estimated quite tightly, with quite narrow credible intervals. The confidence intervals for the autocorrelation coefficients ρ_A and ρ_d, and also the interval for the elasticity of substitution is somewhat larger, reflecting the results for the strength of identification. Elasticity of substitution σ is however estimated to be significantly below one, so supply and demand are complements in the goods market matching function. This can be also seen in Figure 2.3 that plots posterior distributions for estimated parameters.

Table 2.1: Parameter estimates for model with shocks to γ_A, z, ζ_d

	Prior			Posterior		
	distribution	mean	st.dev.	mode	mean	90 % HPD interval
ρ_d	Beta	0.8	0.1	0.9466	0.9469	[0.9139,0.9832]
ρ_z	Beta	0.8	0.1	0.9964	0.9958	[0.9930,0.9987]
ρ_A	Beta	0.6	0.2	0.3202	0.3100	[0.1006,0.5135]
st.dev. ε_d	Inverse Gamma	0.01	2	0.0081	0.0086	[0.0071,0.0100]
st.dev. ε_z	Inverse Gamma	0.01	2	0.0042	0.0042	[0.0038,0.0047]
st.dev. ε_A	Inverse Gamma	0.01	2	0.0020	0.0020	[0.0016,0.0025]
σ	Gamma	1	1	0.4882	0.5157	[0.3799,0.6397]
	Log data density 1413.36					

Figure 2.3: Prior and posterior distributions, model with shocks to γ_A, z, ζ_d



2.3.3 Simulation

Table 2.2 shows the main business cycle moments of the model with inventories. Based on the results it is clear that adding goods market frictions improves the ability of the model to replicate the behavior of labor market observed in the U.S. data. Unemployment, vacancies and vacancy-unemployment ratio are about twice as volatile in the extended model as in the model with labor search only in Table 1.5. The model is also able to match other facts from U.S. data - procyclical inventories, countercyclical inventories-sales ratio, and sales less volatile than output.

The variance decomposition shows that demand shocks are therefore important factors in explaining the behavior of unemployment over the business cycle. They account for more than 90% of short run variance of the labor market variables. And even in the long run, about 60% of the fluctuations in the labor market variables in the model

Table 2.2: Summary statistics, goods and labor search model with inventories

	US data			GLS model		
	st.dev.	corr$(\cdot, y)$	corr$(\cdot, Y)$	st.dev.	corr$(\cdot, y)$	corr$(\cdot, Y)$
y	0.014	1.00	0.87	0.015	1.00	0.99
θ	0.262	0.62	0.88	0.100	0.69	0.76
V	0.140	0.66	0.89	0.060	0.64	0.69
U	0.127	-0.55	-0.83	0.045	-0.66	-0.74
Y	0.021	0.87	1.00	0.017	0.99	1.00
S	0.018	0.58	0.94	0.015	0.89	0.92
I	0.018	-0.12	0.54	0.011	0.52	0.45
I/S	0.017	-0.73	-0.41	0.016	-0.50	-0.57

are due to shocks technology to preferences; the remaining 40% are due to transitory shocks to technology. For labor productivity the decomposition is the opposite, with about 60% of long run variance being attributed to transitory technology shocks. Similarly the inventory-sales ratio is affected mainly by preference shocks in the short run, but in the long run transitory technology shocks play the main role.

Table 2.3: Conditional Variance Decomposition

	Q1		Q4		Q40		Q100	
	z	ζ_d	z	ζ_d	z	ζ_d	z	ζ_d
θ	6.87	93.11	9.57	90.42	33.4	66.6	41.92	58.08
Y	35.96	56.98	19.62	76.64	18.8	78.37	20.42	76.8
Y/N	41.07	51.05	26.63	68.64	30.89	65.66	34.78	61.96
I/S	18.92	80.96	29.94	70.01	68.35	31.63	75.71	24.28

2.3.4 Technology versus Preference Shocks

Preference shock to disutility from search ζ_d is specific to the environment with goods market friction, but a number of papers include other preference shocks, that affect marginal utility of consumption (e.g, Rabanal & Rubio-Ramirez, 2005, Khan & Thomas, 2007a) or marginal utility of leisure (e.g. Del Negro & Schorfheide, 2008, Hagedorn & Manovskii, 2011, Rios-Rull, Schorfheide, Fuentes-Albero, Kryshko, & Santaeulalia-Llopis, 2012), or an intertemporal preference shock to the discount factor (e.g Justiniano & Michelacci, 2011, Justiniano, Primiceri, & Tambalotti, 2010, Schmitt-Grohé & Uribe, 2012). Chapter 1 demonstrates that in the environment with goods market frictions a shock to marginal utility of consumption by itself can actually generate fluctuations in labor productivity, output, employment and labor market tightness very similar to those in the U.S. data when elasticity of substitution of the goods market function is $\sigma = 2$. The elasticity of substitution in that model was however not obtained by estimation. I thus reestimate the model with inventories, now with technology shocks z and A and with preference shock ζ_c. The observables are again the labor productivity growth rate γ_y and the inventory to sales ratio ι. The reason for the choice of observables is that a preference shock ζ_c has a similar effect on inventory to sales ratio as the preference shock ζ_d, it leads to a fall in inventories relative to sales.

The results of estimation are shown in Table 2.4 and the implied business cycle properties of the model are in Table 2.5. The estimate for the elasticity of substitution is close to the value obtained in model with preference shocks ζ_d, but the overall performance of the model is now somewhat closer to the data. With shock to the marginal utility of consumption present in the model, fluctuations of the labor market tightness and unemployment are about 60% as large as the ones observed in the U.S. data.

Table 2.4: Parameter estimates for model with shocks to γ_A, z, ζ_c

	Prior			Posterior		
	distribution	mean	st.dev.	mode	mean	90 % HPD interval
ρ_c	Beta	0.8	0.1	0.9354	0.9346	[0.8958,0.9754]
ρ_z	Beta	0.8	0.1	0.9961	0.9956	[0.9928,0.9986]
ρ_A	Beta	0.6	0.2	0.3515	0.3267	[0.1101,0.5278]
st.dev. ε_c	Inverse Gamma	0.01	2	0.0081	0.0083	[0.0072,0.0093]
st.dev. ε_z	Inverse Gamma	0.01	2	0.0044	0.0045	[0.0040,0.0049]
st.dev. ε_A	Inverse Gamma	0.01	2	0.0020	0.0021	[0.0016,0.0025]
σ	Gamma	1	1	0.4490	0.4596	[0.3699,0.5498]
	Log data density 1403.61					

Figure 2.4: Prior and posterior distributions, model with shocks to γ_A, z, ζ_c

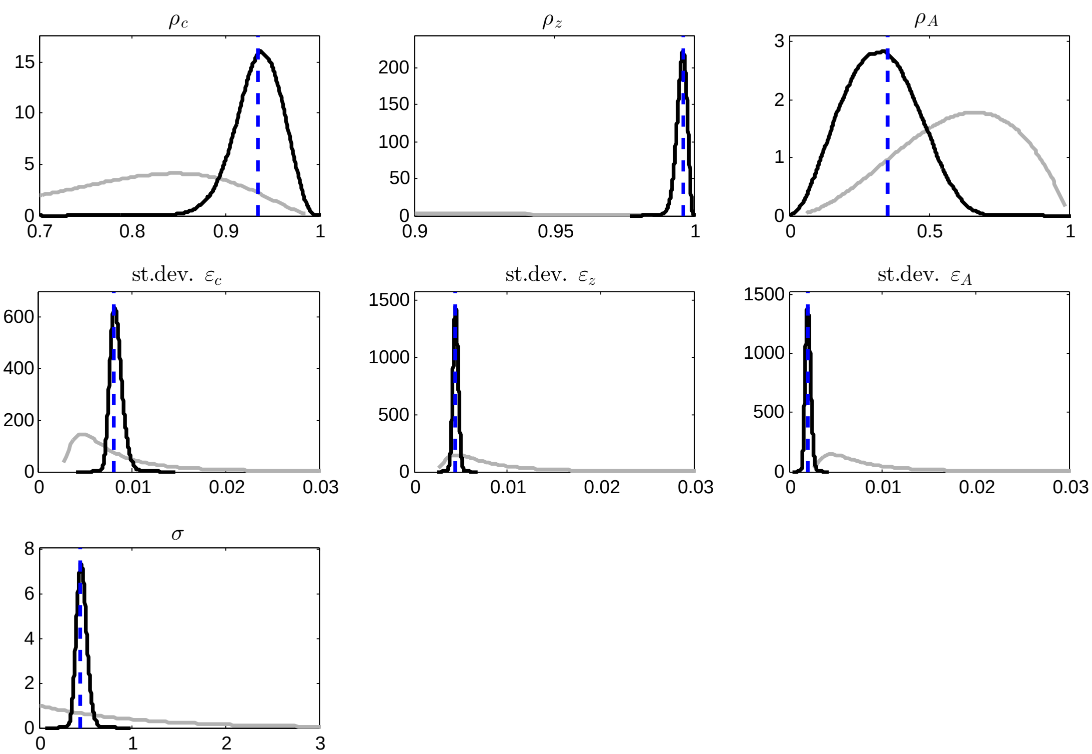


Table 2.5: Summary statistics, goods and labor search model with inventories

	US data			GLS model		
	st.dev.	corr$(\cdot, y)$	corr$(\cdot, Y)$	st.dev.	corr$(\cdot, y)$	corr$(\cdot, Y)$
y	0.014	1.00	0.87	0.0150	1.00	0.98
θ	0.262	0.62	0.88	0.1621	0.66	0.77
V	0.140	0.66	0.89	0.0995	0.63	0.71
U	0.127	-0.55	-0.83	0.0740	-0.60	-0.74
Y	0.021	0.87	1.00	0.0179	0.98	1.00
S	0.018	0.58	0.94	0.0153	0.86	0.92
I	0.018	-0.12	0.54	0.0120	0.61	0.53
I/S	0.017	-0.73	-0.41	0.0145	-0.40	-0.52

2.4 Conclusion

This paper develops a model with goods and labor market search-matching frictions to analyze the relative importance of technology (supply) shocks and preference (demand) shocks in determining the behavior of unemployment and inventories. With both types of shocks, the response of vacancies and unemployment to changes in measured labor productivity is about twice as large as in the model with labor search only. In addition, demand shocks account for the majority of fluctuations observed in the labor market in the short run, and about half of the fluctuations in the long run. Similarly the behavior of the inventory-sales ratio is mainly driven by demand shocks in the short run and even in the long run they account for about a quarter of the observed fluctuations. The results of this paper show that both the goods market frictions and the demand shocks play an important role in determining the behavior of unemployment over the business cycle. More work however has to be done to further explore the nature of the demand shocks; the model provides an attractive framework to study news, noise, and uncertainty shocks.

Chapter 3

Goods Market Frictions and the Labor Wedge

3.1 Introduction

Business cycle models that incorporate labor market search improve upon frictionless labor market models along several dimensions, as first shown by Andolfatto (1996) and Merz (1995). Shimer (2005) and Shimer (2010) however raises two important issues regarding the labor search and matching model. First, fluctuations in unemployment and vacancies in response to productivity shocks in the calibrated model are much lower than those observed in U.S. data. Second, since search frictions act as adjustment costs, the measured labor wedge (the gap between firm's marginal product of labor and household's marginal rate of substitution) in this model resembles a procyclical labor income tax, contrary to finding in the U.S. data. Thus instead of being able to explain fluctuations in the labor wedge, adding labor market search frictions exacerbates the problem. As a remedy to these issues, Shimer (2010) advocates for wage rigidity, in addition to labor market search frictions, and shows that it helps to explain why unemployment is so volatile and why measured labor wedge resembles a countercyclical tax on labor income. However, the wage rigidity required is that wages of workers in new employment relationships are rigid over the business cycle. Given that the empirical evidence available does not support this claim (see Pissarides, 2009 for a detailed discussion), this solutions is not completely without its own problems. Moreover, Bils, Klenow, and Malin (2014) decompose the labor wedge into product market (price mark-up) and labor market (wage mark-up) components, and argue that product market component is at least as important as labor market component. This implies that sticky wages and labor market matching friction can not fully account for the behavior of the labor wedge, and sticky prices or other frictions that generate countercyclical mark-ups of the product market component deserve more attention in the business cycle research.

In this paper I show that goods market search frictions manifest themselves as a labor wedge. When consumers need to exert effort to purchase goods, value of marginal earnings to a worker are modified by the extra disutility from this search. Similarly, when firm's are only able to sell a fraction of the output supplied to the market because of goods market search frictions, changes in the search effort by consumer's affect the value of the marginal product of the labor. Thus when the economy is subject to technology and preference shocks, a labor wedge equivalent to a countercyclical tax on labor income arises as a consequence of improved ability of firms to sell their goods in expansions, and the lower disutility required per unit of goods purchased by consumers in expansions.

The remainder of the paper is organized as follows. In Section 3.2 the business cycle accounting approach from Chari et al. (2007) is discussed, and used to construct the U.S. labor wedge. Section 3.3 describes the model, Section 3.4 characterizes the equilibrium and shows that the goods market search frictions present themselves as a labor wedge. In Section 3.5 I use Bayesian techniques to parametrize shocks in the model, and then compare the implied business cycle properties of labor wedge generated by the model. Section 3.6 concludes.

3.2 The Labor Wedge

As Chari et al. (2007) point out, in many models mechanisms through which different shocks result in business cycle fluctuations manifest themselves as four wedges in the standard growth model - time varying productivity, labor and investment taxes, and government consumption. This motivates them to propose analysis of these wedges as a method to evaluate which mechanisms are promising in explaining business cycle fluctuations. They show that most of the fluctuations in the postwar period can be accounted for by efficiency and labor wedges, and thus stipulate that it is of particular interest to develop models that are able to replicate the behavior of efficiency and labor wedges observed in data.

The equilibrium in the prototype growth model that Chari et al. (2007) is characterized by a following set of conditions

$$Y = C + K' - (1-\delta_k)K + \tilde{G} \tag{3.2.1}$$

$$Y = \tilde{z}f(K, NH) \tag{3.2.2}$$

$$-U_H(C,H) = (1-\tilde{\tau}_w)\tilde{z}f_L(K,NH)U_C(C,H) \tag{3.2.3}$$

$$(1+\tilde{\tau}_i)U_C(C,H) = \beta\mathbb{E}\Big[\big(\tilde{z}'f_K(K',N'H') + (1+\tilde{\tau}_i')(1-\delta_n)\big)U_C'(C',H')\Big] \tag{3.2.4}$$

where $\tilde{z}$ is the efficiency wedge, $1-\tilde{\tau}_w$ the labor wedge, $\frac{1}{1+\tilde{\tau}_i}$ the investment wedge, and $\tilde{G}$ the government consumption wedge. The first condition is the resource constraint, second one specifies production technology, third is the intratemporal optimality condition for labor, and the last one is the intertemporal optimality condition for capital.

To construct the time series for labor wedge, it's necessary to make assumptions about the functional forms for preferences and technology. Consider the case where utility and production functions are

$$U(C,H) = \log C - \zeta_n \frac{H^{1+\phi}}{1+\phi}$$

and

$$f(K,NH) = K^{1-\bar{\lambda}}(NH)^{\bar{\lambda}}$$

so that (3.2.3) yields the following labor wedge

$$1-\tilde{\tau}_w = \frac{\zeta_n}{\bar{\lambda}}\frac{C}{Y}H^{1+\phi}$$

where $\bar{\lambda}$ the average labor share in the U.S. national income, ζ_n is set to match the average labor wedge of 0.6, and ϕ is in turns chosen to obtain three with Frisch elasticity of labor supply equal to 0.5, 1 and 3. The data used to construct the time series for consumption, output, and hours worked is discussed in Appendix E. The implied time series for labor wedge is shown in Figure 3.1, and its fluctuations around the long run trend, obtain using the Hodrick-Prescott filter with smoothing parameter 1600, are in Figure 3.2. In both figures the grey bands represent the NBER recession dates. The procyclical pattern of the labor wedge $1 - \tilde{\tau}_w$ is clearly visible in both figures; for all three values of Frisch elasticity the labor wedge increases in recessions.

Figure 3.1: U.S. Labor wedge

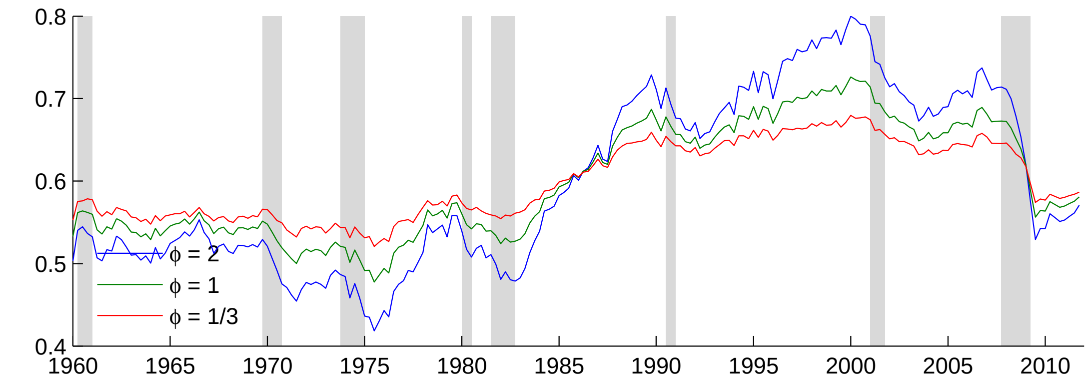
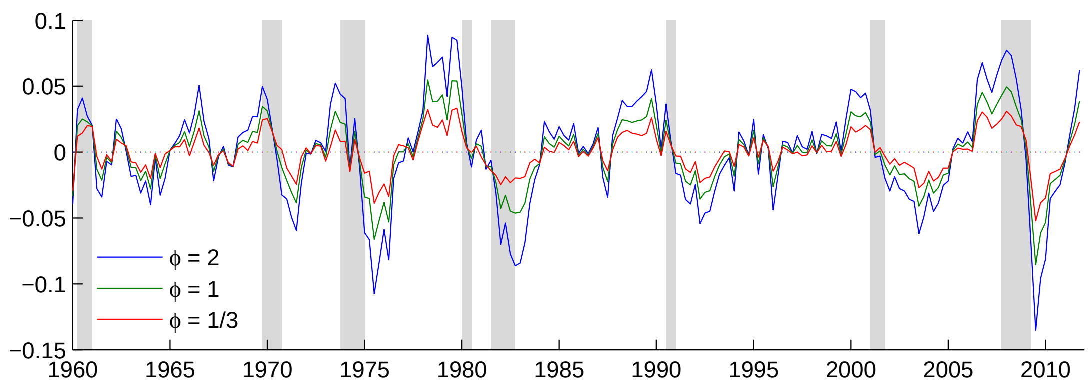


Figure 3.2: U.S. Labor wedge, deviations from the HP trend



3.3 Model

In this section I describe the model with frictional labor and goods markets which is afterwards used to analyze the behavior of the labor wedge. There is a measure one of identical households, each consisting of a continuum of measure one of workers. Workers have to search in the market for jobs and also for consumption goods. There is also a continuum of firms with measure one, each using capital and labor services to produce goods. I assume that workers cannot quit and that job destruction is exogenous. Consumption goods produced by firms are sold under competitive search in submarkets, each indexed by price and tightness (p, Q). Search in labor market is undirected and wage in each match is determined by Nash bargaining between worker and firm.

The aggregate state of the economy is $\mathbf{S} = (z, \zeta, K, I, N)$, where z is the current level of total factor productivity, ζ the current shock to preference parameters, K the aggregate capital stock, I the stock of inventories, and N the measure of employed workers after separations take place.

Labor Market

As in standard labor search model, search in the labor market is not directed, number of matches is given by an aggregate constant returns to scale matching function $m^L(U, V)$ where U are unemployed workers and V are the vacancies posted by firms. I denote by $\theta = \frac{V}{U}$ tightness of the market, and by $\pi^u(\theta) = m^L(1, \theta)$ the probability for an unemployed worker to be hired, and by $\pi^v(\theta) = m^L(1/\theta, 1)$ the rate at which a recruiter hires workers. If a worker and a recruiter meet, wage w and hours worked H is set in every period as a solution to the asymmetric Nash bargaining problem that splits the surplus of the match

$$(w(\mathbf{S}), H(\mathbf{S})) = \operatorname*{argmax}_{\hat{w}, \hat{H}} \hat{W}_n(\hat{w}, \hat{H})^{\mu} \hat{\Omega}_n(\hat{w}, \hat{H})^{1-\mu} \tag{3.3.1}$$

where $\hat{W}_n(\hat{w}, \hat{h})$ is household's marginal value of a worker employed under a contract requiring arbitrary hours worked $\hat{H}$ at arbitrary wage $\hat{w}$ in the current period and equilibrium hours H at equilibrium wage w thereafter, until the job is hit by the separation shock δ_n. Similarly $\hat{\Omega}_n(\hat{w}, \hat{H})$ is firm's marginal value of an employed worker under a contract requiring arbitrary hours $\hat{H}$ at arbitrary wage $\hat{w}$ in the current period and equilibrium hours H and equilibrium wage w thereafter, until the job is hit by the separation shock δ_n.

Goods Market

Acquisition of consumption goods requires active search effort on the side of the consumer to find the goods to be purchased, and I use the competitive search mechanism (Moen, 1997) to model the frictions in the goods market. Goods market is thus divided into submarkets, firm and household can choose in which submarket to search, and the maches in each submarket are determined by the same constant returns to scale matching function $m^G(D, TX)$ with elasticity of substitution σ_{DX}. Here D is aggregate search effort of all consumers in that particular submarket, T the measure of firms selling in that particular submarket and X is the quantity of goods sold per firm. Submarkets are indexed by (p, Q) where p is the price of consumption good in terms of the shares and $Q = \frac{T}{D}$ is the tightness of the submarket. The amount of goods acquired per unit of search effort by household's shopper is

$$\psi^d(Q, X) = m^G(1, QX)$$

and amount of output successfully sold by a firm trying to sell x goods in submarket (p, Q), where the total amount of goods sold by all firms is X is

$$\psi^x(Q, X)x = m^G\Big(\frac{1}{QX}, 1\Big)x = \frac{1}{X}\frac{\psi^d(Q, X)}{Q}x$$

Household

Households are extended families, consisting of a measure one of workers as in Merz (1995). All workers are ex-ante identical and have preferences

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t \mathrm{u}(c_t, d_t, e_t, h_t, \zeta_t)$$

where c_t is consumption, d_t search effort in goods market, $e_t \in \{0, 1\}$ employment status and ζ_t is the preference shock affecting the marginal disutility of search for consumption good.

In the recursive formulation of the households problem, the individual state of the household is given by amount of shares held s and number of members of the household employed after separations take place n. Household decides about goods market search effort of employed and unemployed workers d_e, d_u, consumption allocation c_e, c_u, and about share holdings for next period s'. Each member also individually decides in which submarket (p, Q) to search for consumption goods, and directs the search to the submarket that delivers the biggest contribution to the utility of the household. I incorporate this through a constraint in the problem of a firm which posts priceq and decides about quantity sold. In addition, since in equilibrium only one market is going to be active, in the household's problem price of goods and goods market tightness appear as given functions of state $p(\mathbf{S}), Q(\mathbf{S})$.

Each employed worker receives before tax wage $w(\mathbf{S})$ and works $H(\mathbf{S})$ hours, each unemployed worker receives unemployment benefits $p(\mathbf{S})b$; labor income and unemployment benefits are taxed at rate τ_w and the household receives transfers $t(\mathbf{S})$. Taking prices $p(\mathbf{S})$, $w(\mathbf{S})$, hours worked by each employed worker $H(\mathbf{S})$, and dividends $R(\mathbf{S})$ as given, the household then faces a budget constraint

$$p(\mathbf{S})\big(nc_e + (1-n)c_u\big) + s' = s(1+R(\mathbf{S})) + (1-\tau_w)\big(nH(\mathbf{S})w(\mathbf{S}) + (1-n)p(\mathbf{S})b\big) + t(\mathbf{S}) \tag{3.3.2}$$

Search in goods market imposes a constraint

$$nc_e + (1-n)c_u = \big(nd_e + (1-n)d_u\big)\psi^d(Q(\mathbf{S}), X(\mathbf{S})) \tag{3.3.3}$$

and the search in labor market constraint

$$n' = (1-\delta_n)n + (1-n)\pi^u(\theta(\mathbf{S})) \tag{3.3.4}$$

where $\pi^u(\theta)$, $\psi^d(Q, X)$ are probabilities for an individual to be matched in labor and in goods market that households takes as given functions of $\theta(\mathbf{S})$, $Q(\mathbf{S})$ and $X(\mathbf{S})$.

Since the optimal allocation of consumption and search effort among family members in each period solves the problem

$$U(c, d, n, h, \zeta) = \max_{c_e, c_u, d_e, d_u} \Big\{ n\mathrm{u}(c_e, d_e, 1, h, \zeta) + (1-n)\mathrm{u}(c_u, d_u, 0, 0, \zeta) \Big\} \tag{3.3.5}$$

subject to

$$nc_e + (1-n)c_u = c$$

$$nd_e + (1-n)d_u = d$$

where c is the total amount of consumption goods available to household and d is the overall search effort, I can formally set up the household's problem in which it acts as if it had preferences with utility function $U(c, d, n, h, \zeta)$. In the recursive formulation, the problem of the household is then

$$W(s, n; \mathbf{S}) = \max_{c,d,s'} \Big\{ U(c, d, n, h, \zeta) + \beta \mathbb{E} W(s', n'; \mathbf{S}') \Big\} \tag{3.3.6}$$

subject to constraints (3.3.2), (3.3.3), (3.3.4), (3.3.5) and $\mathbf{S}' = G(\mathbf{S})$.

Firm

The individual state of a firm is the capital stock k, stock of inventories i, and the number of workers employed n. Each firm chooses in which submarket (p, Q) to sell the goods, and simultaneously also how many vacancies v to open and how much of the production to retain and use to add to the capital stock. The production of a firm is given by function $zf(k, l)$ where z is the productivity and l are the total hours worked in production. The amount of goods x that the firm can potentially sell is

$$x = zf(k, nh - \chi v) - \kappa(v) - k' + (1 - \delta_k)k + i$$

where $f_l > 0$, $f_{ll} \leq 0$ and $\kappa_v \geq 0$, $\kappa_{vv} \geq 0$ which can be interpreted as a case where some of the workers act as recruiters and thus χv hours worked are diverted from the production process to hiring, and in addition $\kappa(v)$ costs in terms of goods are incurred for vacancy posting. Each vacancy attracts $\pi^v(\theta)$ new workers.

If the firm decides to sell it's output x in the (p, Q) submarket, where the aggregate amount of goods being sold is X, then the actual amount of goods for which the firm will be able to find a customer and sell is given by

$$x\psi^x(Q, X) = \frac{x}{X}\frac{\psi^d(Q, X)}{Q}$$

The firm can store goods that are not sold, in an attempt to sell them in the next period. Let i' be the amount of goods carried over to the next period, given by

$$i' = (1 - \delta_i)(1 - \psi^x(Q, X(\mathbf{S})))x$$

where $\delta_i \in (0, 1)$ captures the loss of value due to obsoleteness, the fact that some goods will not be demanded at all in the future, storage costs, and the fact that services can not be stored.

As discussed above in section with household's problem, the firm needs to take into account the constraint guaranteeing shoppers in the (p, Q) submarket at least the equilibrium value of search $W_d^*(\mathbf{S})$. Let $M(\mathbf{S})$ denote the value of unit of shares in terms of utility, then

$$W_d(\mathbf{S}) = U_d(\mathbf{S}) + (U_c(\mathbf{S}) - pM(\mathbf{S}))\psi^d(Q, X(\mathbf{S}))$$

is the value to the household of the marginal search effort in the (p, Q) submarket. Finally, let $m(\mathbf{S}, \mathbf{S}')$ be the stochastic discount factor used to discount future profits.

To summarize, the problem that a firm solves is

$$\Omega(k,i,n;\mathbf{S}) = \max_{v,p,Q,i}\left\{p\psi^x(Q,X(\mathbf{S}))x - (1+\tau_f)w(\mathbf{S})H(\mathbf{S})n + E\big[m(\mathbf{S},\mathbf{S}')\Omega(k',i',n';\mathbf{S}')\big]\right\} \tag{3.3.7}$$

subject to

$W_d^* = U_d(\mathbf{S}) + (U_c(\mathbf{S}) - pM(\mathbf{S}))\psi^d(Q,X(\mathbf{S}))$

$x = zf(k, nH(\mathbf{S}) - \chi v) - \kappa(v) - k' + (1-\delta_k)k + i$

$i' = (1-\delta_i)(1-\psi^x(Q,X(\mathbf{S})))x$

$n' = (1-\delta_n)n + \pi^v(\theta(\mathbf{S}))v$

$\mathbf{S}' = G(\mathbf{S})$

Government

Government's budget is assumed to be balanced in each period, thus total tax revenues are equal to total government expenditures

$$\tau_w\big(nH(\mathbf{S})w(\mathbf{S}) + (1-n)p(\mathbf{S})b\big) + \tau_f nH(\mathbf{S})w(\mathbf{S}) = (1-n)p(\mathbf{S})b + t(\mathbf{S}) \tag{3.3.8}$$

Equilibrium

Given government's policy (τ_w, τ_f, b) an equilibrium in this economy is defined as follows.

Definition 3. *An equilibrium consists of household's value function and policy functions $(W, g^c, g^d, g^{s'})$; firm's value and policy functions $(\Omega, g^v, g^{k'}, g^p, Q)$; aggregate allocation (X, C, D, V, H), tightness (Q, θ), prices (p, w), dividends R, transfers t, law of motion G, all as functions of S; such that*

1. *$(W, g^c g^{\cdot} d, g^{s'})$ are the solution to (3.3.6)*
2. *$(\Omega, g^v, g^{k'}, g^p, g^Q)$ are the solution to (3.3.7)*
3. *Household and firm are representative*
4. *Wage and hours worked $w(\mathbf{S}), h(\mathbf{S})$ solve the Nash bargaining problem (3.3.1)*
5. *Government's budget constraint (3.3.8) is satisfied*
6. *Goods market tightness is $Q(\mathbf{S}) = \frac{1}{D(\mathbf{S})}$; labor market tightness $\theta(\mathbf{S}) = \frac{V(\mathbf{S})}{1-N}$*

3.4 Equilibrium Characterization

To obtain conditions that determine the dynamics of model I first derive the optimality conditions for the household and the firm and then use them to obtain the solution for the Nash bargaining problem. This allows to characterize the behavior of the six main variables in the model (K, I, N, H, Q, θ).

To avoid the notational clutter in what follows I drop the arguments of functions, use g_A to denote derivative of function g with respect to A, and g' to denote value of function g in the next period. I also use ϵ_B^A for elasticity of A with respect to B.

3.4.1 Household's Optimality Conditions

From the first order conditions for c and d we get for the value of the marginal unit of income

$$\lambda_1 = \frac{1}{p}\Big(U_c + \frac{U_d}{\psi^d}\Big)$$

and applying the envelope theorem we get the following expression for the marginal value of a worker employed under a contract with equilibrium hours of work H and equilibrium wage w

$$W_n = U_n + (1-\tau_w)\Big(\frac{w}{p}H - b\Big)\Big(U_c + \frac{U_d}{\psi^d}\Big) + (1-\delta_n-\pi^u)\beta\mathbb{E}W_n' \tag{3.4.1}$$

From the first order conditions for s' and using envelope theorem for s

$$\lambda_1 = \beta\mathbb{E}[\lambda_1'(1+R')]$$

Plugging in for λ_1 yields the following Euler equation equalizing the cost of increasing saving by a marginal unit and the return from this marginal savings

$$\frac{1}{p}\Big(U_c + \frac{U_d}{\psi^d}\Big) = \beta\mathbb{E}\left[(1+R')\frac{1}{p'}\Big(U_c' + \frac{U_d'}{(\psi^d)'}\Big)\right] \tag{3.4.2}$$

The left hand side corresponds to the utility cost of extra unit of savings: the household could have instead purchased $\frac{1}{p}$ units of good which require utility cost $\frac{U_d}{\psi^d}$ per unit of good because of the search friction, and enjoyed U_c extra utility per unit of good. The right hand side corresponds to the utility benefit of extra unit of savings: the $1+R$ monetary flow in the next period can be used to purchase extra consumption in the next period. It will be convenient to denote by $M(\mathbf{S})$ the expected discounted utility from marginal unit of share holdings

$$M = \beta \mathbb{E}\Big[(1+R')\frac{1}{p'}\Big(U_c' + \frac{U_d'}{(\psi^d)'}\Big)\Big] \tag{3.4.3}$$

The above intertemporal optimality condition thus states that $\lambda_1 = M$.

3.4.2 Firm's Optimality Conditions

Since the household is representative adding the full set of Arrow securities would not affect the allocation, and I can use standard complete markets pricing approach to value the firm. Thus we have for the stochastic discount factor

$$m(\mathbf{S},\mathbf{S}') = \beta \frac{p(\mathbf{S})}{p(\mathbf{S}')} \frac{U_c(\mathbf{S}') + \frac{U_d(\mathbf{S}')}{\psi^d(Q(\mathbf{S}'))}}{U_c(\mathbf{S}) + \frac{U_d(\mathbf{S})}{\psi^d(Q(\mathbf{S}))}} \tag{3.4.4}$$

where, with slight abuse of notation, $U_c(\mathbf{S}) = \frac{\partial}{\partial c} U_c(C(\mathbf{S}), D(\mathbf{S}), N(\mathbf{S}), \zeta)$ and similarly $U_d(\mathbf{S}) = \frac{\partial}{\partial c} U_d(C(\mathbf{S}), D(\mathbf{S}), N(\mathbf{S}), \zeta)$.

From the first order condition for Q we get that the equilibrium price of the consumption good satisfies

$$p = \epsilon_Q^{\psi^d} \frac{U_c}{M} + (1 - \epsilon_Q^{\psi^d})(1-\delta_i)\mathbb{E}[m\Omega_i'] \tag{3.4.5}$$

where $\epsilon_Q^{\psi^d} = \frac{\partial \log \psi^d}{\partial \log \psi^d}$, and the value of the marginal unit of inventories satisfies

$$\Omega_i = \psi^x p + (1-\psi^x)(1-\delta_i)\mathbb{E}[m\Omega_i'] \tag{3.4.6}$$

Next, applying the envelope theorem for k and combining it with the first order condition for k' yields the intertemporal condition for optimal capital accumulation

$$\Omega_i = \mathbb{E}\big[m\Omega_i'(z' f_k' + 1 - \delta_k)\big] \tag{3.4.7}$$

Then, applying the envelope theorem for n and combining it with the first order condition for v and the price equation (3.4.5) to get for the marginal value of a worker

$$\Omega_n = \Big(z f_l H + \frac{1-\delta_n}{\pi^v}(\chi z f_l + \kappa_v)\Big)\Omega_i - (1+\tau_f) w H \tag{3.4.8}$$

Finally, by plugging Ω_n into the first order condition for v we obtain the job creation condition

$$(\chi z f_l + \kappa_v)\Omega_i = \pi^v \mathbb{E}\Big[m\Big(\Big(z' f_l' H' + (\chi z' f_l' + \kappa_v')\frac{1-\delta_n}{(\pi^v)'}\Big)\Omega_i' - (1+\tau_f)w' H'\Big)\Big] \tag{3.4.9}$$

3.4.3 Goods Market and Capital Accumulation

From (3.4.5), combined with (3.4.2) and (3.4.3), after eliminating p and M one can obtain the following condition

$$-U_d = (1-\epsilon_Q^{\psi^d})\psi^d\left[U_c - (1-\delta_i)\beta\mathbb{E}\left[\left(U_c' + \frac{U_d'}{(\psi^d)'}\right)(\Omega_i^r)'\right]\right] \tag{3.4.10}$$

which states that the marginal cost and the marginal benefit of search effort in the goods market are equalized.

By plugging the expression for the stochastic discount factor (3.4.4) into the optimality condition from firm's problem (3.4.7) we can derive the following Euler equation for optimal capital accumulation

$$\Omega_i^r\left(U_c + \frac{U_d}{\psi^d}\right) = \beta\mathbb{E}\left[(\Omega_i^r)'(z'f_k' + 1 - \delta_k)\left(U_c' + \frac{U_d'}{(\psi^d)'}\right)\right] \tag{3.4.11}$$

Using (3.4.4) and the value of the marginal unit of inventories from firm's problem, one can also obtain that in equilibrium

$$\Omega_i^r = \psi^x + (1-\psi^x)\beta(1-\delta_i)\mathbb{E}\left[\frac{U_c' + \frac{U_d'}{(\psi^d)'}}{U_c + \frac{U_d}{\psi^d}}(\Omega_i^r)'\right] \tag{3.4.12}$$

3.4.4 Labor Market and Employment Determination

Under Nash bargaining protocol, wage w and hours worked H are jointly determined as a solution to the following problem

$$(w(\mathbf{S}), H(\mathbf{S})) = \underset{\hat{w},\hat{H}}{\operatorname{argmax}}\, \hat{W}_n(\hat{w},\hat{H})^{\mu}\hat{\Omega}_n(\hat{w},\hat{H})^{1-\mu}$$

where $\hat{W}_n(\hat{w},\hat{H})$ and $\hat{\Omega}_n(\hat{w},\hat{H})$ are values, to household and firm, of a marginal worker employed under a contract requiring arbitrary hours worked $\hat{H}$ at arbitrary wage $\hat{w}$ in the current period and equilibrium hours H at equilibrium wage w thereafter, until the job is hit by the separation shock δ_n.

By considering a household with n members employed for equilibrium wage w and working equilibrium hours H, and ν members employed for arbitrary wage $\hat{w}$ and working arbitrary hours $\hat{H}$ in the current period and equilibrium wage w and equilibrium hours H thereafter, until the they are hit by the separation shock δ_n, and taking the limit as $\nu \to 0$, we can obtain the value of a marginal member of the household employed for this households

$$\hat{W}_n(\hat{w}, \hat{H}) = U_n(\hat{H}) - U_n(H) + (1 - \tau_w)\frac{\hat{w}\hat{H} - wH}{p}\Big(U_c + \frac{U_d}{\psi^d}\Big) + W_n \tag{3.4.13}$$

By considering a firm that employs n workers employed at equilibrium wage w and equilibrium hours H, and ν workers employed at arbitrary wage $\hat{w}$ and arbitrary hours $\hat{H}$ in the current period, and equilibrium w and H thereafter, and taking the limit as $\nu \to 0$, we can obtain the value of an extra worker for the firm

$$\hat{\Omega}_n(\hat{w}, \hat{H}) = zf_l(\hat{H} - H)\Omega_i + (1 + \tau_f)(wH - \hat{w}\hat{H}) + \Omega_n \tag{3.4.14}$$

The Nash bargaining problem is thus

$$(w(\mathbf{S}), H(\mathbf{S})) = \underset{\hat{w},\hat{H}}{\operatorname{argmax}}\, \hat{W}_n(\hat{w}, \hat{H})^{\mu}\hat{\Omega}_n(\hat{w}, \hat{H})^{1-\mu}$$

subject to (3.4.13) and (3.4.14). The first order condition for w yields a sharing rule

$$W_n = \frac{\mu}{1-\mu}\left(U_c + \frac{U_d}{\psi^d}\right)\frac{1-\tau_w}{1+\tau_f}\frac{\Omega_n}{p} \tag{3.4.15}$$

or $\frac{W_n}{p\lambda_1} = \mu\mathcal{S}$ where $\lambda_1 = \frac{1}{p}\big(U_c + \frac{U_d}{\psi^d}\big)$ is the marginal value of wealth for the household and $\mathcal{S} = \frac{\Omega_n}{p} + \frac{W_n}{p\lambda_1}$ is the total surplus of the match.

From the first order condition for h we get, using the first order condition for w, the following condition for hours worked

$$-U_{n,h} = \frac{1-\tau_w}{1+\tau_f}\Omega_i^r zf_l\Big(U_c + \frac{U_d}{\psi^d}\Big) \tag{3.4.16}$$

To derive the wage equation first plug W_n from the sharing rule (3.4.15) into (3.4.1), use stochastic discount factor (3.4.4), and the optimality condition for firm (3.4.8) and (3.4.9) which after a little bit of algebra yields the equation for wage bill per worker

$$\frac{wH}{p} = \mu\frac{1}{1+\tau_f}\Big(zf_lH + (\chi zf_l + \kappa_v)\theta\Big)\Omega_i^r + (1-\mu)\Bigg(b - \frac{1}{1-\tau_w}\frac{U_n}{U_c + \frac{U_d}{\psi^d}}\Bigg) \tag{3.4.17}$$

Similarly to other search-matching models with Nash bargaining, wage is a weighted average of the value of marginal product of a worker enhanced by the vacancy cost savings, and the marginal rate of substitution between leisure and consumption.

Finally, to get a stochastic difference equation that characterizes the labor market plug in for w' from (3.4.17) into the job creation equation (3.4.9), and use stochastic discount factor (3.4.4) to get

$$\begin{aligned}
&\frac{1}{\pi^v}(\chi z f_l + \kappa_v)\Omega_i^r\left(U_c + \frac{U_d}{\psi^d}\right) \\
&= \beta\mathbb{E}\Bigg[\left[(1-\mu)z' f_l' H' + \left(\frac{1-\delta_n}{(\pi^v)'} - \mu\theta'\right)(\chi z' f_l' + \kappa_v')\right](\Omega_i^r)'\left(U_c' + \frac{U_d'}{(\psi^d)'}\right) \\
&\qquad\qquad - (1-\mu)\left[(1+\tau_f)b\left(U_c' + \frac{U_d'}{(\psi^d)'}\right) - \frac{1+\tau_f}{1-\tau_w}U_n'\right]\Bigg]
\end{aligned} \tag{3.4.18}$$

3.4.5 Efficiency

The results so far are summarized by following proposition.

Proposition 6. *Equilibrium market tightness (Q, θ) and allocation (H, K, I, N) is given by the system of equations (3.4.10), (3.4.11), (3.4.12), (3.4.16), (3.4.18), with*

$$N' = (1-\delta_n)N + (1-N)\pi^u \tag{3.4.19}$$

$$I' = (1-\delta_i)(1-\psi^x)X \tag{3.4.20}$$

and with (C, X, V, D) eliminated using constraints

$$\begin{aligned}
C &= \psi^d/Q \\
X &= zf - \kappa + (1-\delta_k)K - K' + I \\
V &= \theta(1-N) \\
Q &= T/D
\end{aligned}$$

The first one of the seven equations in Proposition 6 is the optimality conditions for goods market with competitive search, in which the effect of the search effort by a consumer on the ability of other participants in the market to trade successfully is internalized. The second condition is an Euler equation for capital accumulation, where the left hand side represents the cost of marginal unit of output allocated into investment, in the form of foregone utility from consumption, and the right hand side is the benefit of this marginal investment unit in the form of extra utility derived from extra consumption in the next period. Utility gain from consumption on both sides is adjusted for the utility cost incurred due to an increase in search effort needed to purchase these goods. The next equation characterizes the value of the inventory, which depends on the ability of the firm to successfully sell it in the current period, and on its depreciated value when the inventory is not sold in the current period. The fourth condition the intratemporal optimality conditions for hours worked in equilibrium showing that hours worked optimally equate the utility cots of an extra hour of work with its benefit, which is the utility gain from consumption of goods produced and purchased, adjusted for the utility cost incurred due to extra search needed to purchase these goods. The fifth equation is the counterpart of the stochastic first order difference equation for labor market tightness θ in the basic labor search model. This condition equates the cost of hiring a worker in terms of utility (fewer goods sold and thus also consumed), with the value of an extra worker hired in terms of utility (increased production and hiring cost saved which both allow to increase consumption in future, adjusted for the value of foregone leisure).

The efficient allocation in this economy is defined as an allocation chosen by a social planner facing the search-matching technological restrictions in the labor and goods markets

Definition 4. *An allocation is efficient if it solves*

$$\mathcal{W}(z,\zeta,K,I,N) = \max_{C,D,H,X,V,K'} \Big\{ U(C,D,N,H,\zeta) + \beta E \mathcal{W}(z',\zeta',K',I',N') \Big\}$$

subject to

$$C = m^G(D,TX)$$

$$X = zf(K,NH-\chi V) - \kappa(V) + (1-\delta_k)K - K' + I$$

$$I' = (1-\delta_i)(X - m^G(D,TX))$$

$$N' = (1-\delta_n)N + m^L(U,V)$$

Given this definition of efficiency the following proposition establishes condition which guarantees the efficiency of the decentralized economy.

Proposition 7. *If $\tau_w = \tau_f = 0$, $b = 0$ and worker's bargaining power is $\mu = \frac{\partial \log m^L}{\partial \log U}$ equilibrium is efficient.*

This proposition thus implies that the existence of the labor market wedge in this model does not imply inefficiency, as would be the case with wage or price mark-ups due to monopolistic competition or sticky wages and prices.

3.4.6 Labor Wedge in the Model with Goods and Labor Search

Goods market frictions alter the efficiency wedge, but by comparing equilibrium conditions for the model with labor and goods market search in Proposition 6 with those for the prototype RBC model in Section 3.2, in particular the intratemporal conditions for hours worked (3.4.16) and (3.2.3) it is clear that they also affect the labor wedge. If in expansion the disutility associated with obtaining a marginal unit of consumption $\frac{U_d}{\psi^d}$ falls, or if the marginal value of an inventory Ω_i^r increases, the labor wedge in the goods and labor market search model will be more procyclical than the labor wedge in the labor search model from Shimer (2010). To show that this is indeed the case, I now turn to the quantitative analysis of the business cycle properties of the model.

3.5 Quantitative Analysis

3.5.1 Functional forms

I consider the case with following functional forms for preferences and technology.

Assumption 3.

(A1) Utility of an individual worker is given by

$$u(c,d,e,h) = \zeta_c \log c - \zeta_d \frac{d^{1+\varphi}}{1+\varphi} - e\zeta_n \frac{h^{1+\phi}}{1+\phi} - (1-e)\zeta_u$$

(A2) Production function is of Cobb-Douglas form $zAf(k,l) = zAk^{1-\lambda}l^{\lambda}$

(A3) Matching functions have constant elasticity of substitution form

$$m^L(U,V) = B(\gamma U^{\frac{\nu-1}{\nu}} + (1-\gamma)V^{\frac{\nu-1}{\nu}})^{\frac{\nu}{\nu-1}}$$
$$m^G(AD,TX) = \left(\alpha(AD)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(TX)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

(A4) Hiring costs are characterized by $\chi = 1$, $\kappa(V) \equiv 0$

Since preferences are additively separable between consumption, hours worked, and search effort we immediately get $c_e = c_u = c$ and $d_e = d_u = d$. The household thus acts as if it had preferences

$$U(c, d, n, h, \zeta) = \log c - \zeta_d \frac{d^{1+\varphi}}{1+\varphi} - n\zeta_n \frac{h^{1+\phi}}{1+\phi} - (1-n)\zeta_u$$

The technology is subject to temporary shocks to z and to permanent shocks to the stochastic trend A, which grows at rate $\gamma_A' = \frac{A'}{A}$. To guarantee existence of a balanced growth path, the permanent component of the technology also increases efficiency of the search effort by consumers.

The processes for shocks are assumed to be

$$\log \xi' = (1-\rho_\xi)\log \bar{\xi} + \rho_\xi \log \xi + \varepsilon_\xi'$$

where $\xi \in \{\gamma_A, z, \zeta_d\}$ and $\varepsilon_\xi \sim N(0, \eta_\xi^2)$.

3.5.2 Calibration

One period of the model is one quarter, parameter β is chosen to obtain the annual interest rate of 5%. I set $\bar{z}$ to normalize the level of realized consumption $C = 1$ and set λ to target the capital share 0.36. Capital output ratio target is 3.2 as in Shimer (2010), and δ_k is 0.07. I assume a symmetric goods market matching function with $\alpha = 0.5$. Depreciation of inventories is 0.15 for goods and 1 for services, the implied overall quarterly depreciation of inventories is thus 0.83.

For labor market I follow Shimer (2010) by setting $\nu = 0$, $\gamma = 0.5$ which implies a symmetric Cobb-Douglas matching function. I set the value of unemployment benefits b to 0.2 of average labor productivity, target quarterly job separation rate $\delta_n = 0.1$, quarterly job finding rate $\pi^u = 0.733$ and steady state unemployment rate $U = 0.12$. Silva and Toledo (2009) and Hagedorn and Manovskii (2008) argue for average costs associated with recruiting, screening and interviewing needed to hire a new worker around 4% to 5% of new worker's quarterly wages paid. Since an hour of recruitment in the model attracts π^v workers, to get one worker $\frac{1}{\pi^v}$ hours of recruitment are needed. Thus, if w is the wage in the model, the total costs of a hire are $\frac{1}{\pi^v} w = 0.065 \times wH$ and so I target $\pi^v = \frac{1}{0.065H}$. I set τ_w, τ_f to obtain the steady state measured labor wage of 0.6, consistent with U.S. data as discussed in Section 3.2. Given the job finding rate and recruitment rates targeted, since $\frac{\pi^u}{\pi^v} = \theta$ and $\pi^u = B\theta^{1-\gamma}$ the matching efficiency parameter is $B = (\pi^u)^\gamma(\pi^v)^{1-\gamma} = 6.13$.

Parameter ϕ is set to get Frisch elasticity 0.7, and φ is to 0. I calibrate ζ_n so that in the steady state hours worked are $H = 0.3$. To set ζ_u notice that for a given bargaining power μ, value of home production and leisure ζ_u affects wage and through that profits of the firms, hiring, labor tightness θ, and also U. I thus proceed as Shimer (2010), set $\mu = \gamma$ and calibrate ζ_u to match the above mentioned target unemployment rate. I set $\bar{\zeta}_c = 1$ and calibrate $\bar{\zeta}_d$ to normalize the steady state goods market tightness to $Q = 1$.

3.5.3 Estimation

In order to set the parameters of the processes γ_A, z and ζ_d and the elasticity of substitution for the good market matching function σ, I estimate a log-linearized model to match observed quarterly time series for the growth rate of the measured productivity residual $\gamma_{\hat{z}} = \Delta \log \hat{z}$, the growth rate of per capita output γ_Y, and the growth rate of per capita consumption γ_C. The sample used is 1960Q1-2010Q4, Appendix E describes the data. Table 3.1 shows the prior distributions for estimation, estimated posterior mode obtained by maximizing the log of the posterior distribution, the approximate standard error based on the corresponding Hessian, and also the mean, mode, 10 and 90 percentile of the posterior distribution of the parameters obtained using the random walk Metropolis-Hastings algorithm with four chains and 500000 draws.

Table 3.1: Estimation of the model with shocks to γ_A, z, ζ_d

	Prior			Posterior		
	distribution	mean	st.dev.	mode	mean	90 % HPD interval
ρ_d	Beta	0.8	0.1	0.9914	0.9893	[0.9821,0.9968]
ρ_z	Beta	0.8	0.1	0.9205	0.9237	[0.8888,0.9591]
ρ_A	Beta	0.6	0.2	0.1440	0.1563	[0.0621,0.2458]
st.dev. ε_d	Inverse Gamma	0.01	2	0.0096	0.0101	[0.0081,0.0121]
st.dev. ε_z	Inverse Gamma	0.01	2	0.0041	0.0043	[0.0036,0.0050]
st.dev. ε_A	Inverse Gamma	0.01	2	0.0060	0.0060	[0.0055,0.0065]
σ	Gamma	1	1	0.3378	0.3861	[0.2305,0.5309]

3.5.4 Simulation

Table 3.2 compares the average standard deviations and correlations of the main variables in 1000 simulations of the model with their counterparts in U.S. data. All variables are in logs, HP filtered with parameter $\lambda = 1600$. The statistics for labor wedge are presented for both the representative agent wedge, and for the intensive margin wedge which allows to distinguish the hours per worker and the employment components. The representative agent wedge is defined as

$$RAW = \frac{MRS}{MP_L} = \frac{\zeta_n (NH)^{\phi} / \frac{1}{C}}{\bar{\lambda} \frac{Y}{NH}}$$

and the intensive margin wedge as

$$IMW = \frac{MRS_H}{MPN_H} = \frac{N \zeta_n H^{\phi} / \frac{1}{C}}{\bar{\lambda} \frac{Y}{NH} N}$$

Thus in the representative margin wedge the marginal rate of substitution of consumption for leisure is based on hours per capita, where as in the intensive margin wedge it is based on the hours per worker. Similarly, the marginal product in the representative margin wedge is the marginal product of an hour, where as in the intensive margin wedge it is marginal product of an hour per worker (see Pescatori & Tasci, 2013 and Bils et al., 2014 for a further discussion on this distinction).

Table 3.2: Summary statistics, based on 1000 simulations of the model

	U.S. data		model	
	st.dev.$(\cdot)$/st.dev.(Y)	corr$(\cdot, Y)$	st.dev.$(\cdot)$/st.dev.(Y)	corr$(\cdot, Y)$
C	0.81	0.88	0.71	0.79
ΔK	3.25	0.93	2.16	0.80
H	0.31	0.79	0.20	0.87
N	0.71	0.83	0.26	0.72
RAW	1.90	0.41	1.01	0.42
IMW	1.05	0.32	0.81	0.22
S	0.90	0.95	0.98	0.99
I	0.86	0.58	0.34	0.76
I/S	0.75	-0.44	0.77	-0.95

In the simulations, both the representative agent labor wedge and the intensive margin labor wedge are somewhat less volatile than in the data. Both are however procyclical, with similar correlation with output as observed in the data.

The historical labor wedge and the smoothed labor wedge obtain in the estimation are plotted in Figure 3.3 and Figure 3.4. The correlation between the representative agent labor wedge in the U.S. data and the one recovered in the estimation is 0.497, for the intensive margin labor wedge this correlation is 0.324. Table 3.3 compares the cyclical properties of the actual labor wedge in the U.S. data and the labor wedge recovered in the estimation.

Figure 3.3: Representative margin wedge

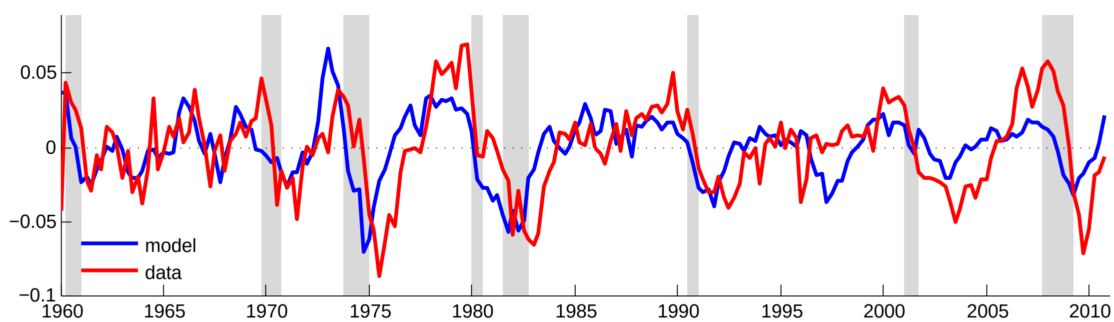


Figure 3.4: Intensive margin wedge

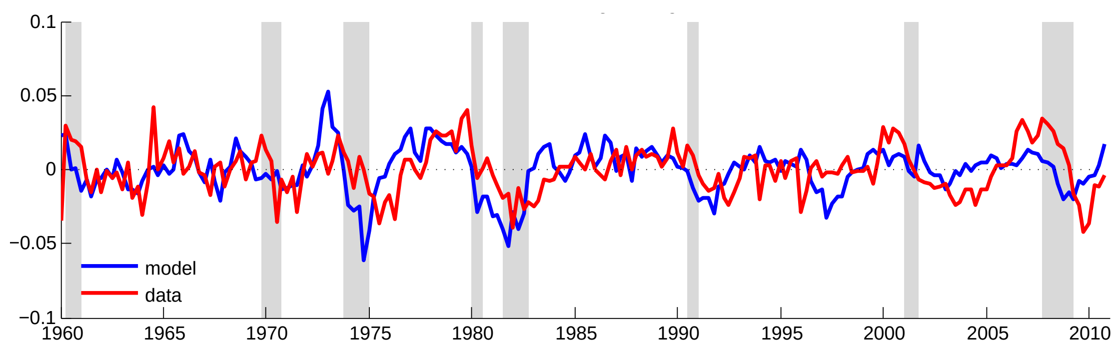


Table 3.2 also shows that even though both consumption and investment are somewhat less volatile in the model compared to the data, they are not very far away. The

Table 3.3: Historical vs. smoothed labor wedge

	U.S. data	model
	Standard deviation relative to output	
RAW	1.897	1.382
IMW	1.044	1.022
	Correlation with output	
RAW	0.71	0.69
IMW	0.53	0.51
	Elasticity with respect to output	
RAW	1.349	0.958
IMW	0.554	0.521

main shortcoming of the model is the implied volatility of employment which is considerably smaller. On the more positive side, even though neither data on inventories nor data on sales was not used as observables in the estimation, the model can replicate the countercyclical inventory-sales ratio, procyclical inventories, and sales which are less volatile than output.

3.6 Conclusions

In this paper, I modify the standard real business cycle model by replacing frictionless labor and goods markets with markets that require search effort of market participants to find a match. I use this model to demonstrate that under the business cycle accounting approach proposed by Chari et al. (2007), search frictions in the goods market manifest themselves as a labor wedge. The model is estimated using Bayesian methods to match U.S. Solow residual, output and consumption growth. Both technology and preference shocks to disutility from search are included in the estimation, to allow for supply and demand side disturbances. In the estimated model with search frictions in both labor and goods markets, firms are more likely to sell goods in expansions due to an increase in demand, and the disutility from search effort required per unit of consumption falls in expansion. As a result there is a larger response of the intensive margin of labor supply and the measured labor wedge resembles a countercyclical tax on labor income. This is in stark contrast to the model in Shimer (2010) where only labor market is subject to search frictions, and the labor wedge resembles a counterfactually procyclical tax on labor income. Since inventories naturally arise in an environment where search frictions prevent output from being sold immediately, the developed model also provides a framework to analyze the behavior of inventories and sales. Even though these are not targeted, the model can successfully match the three main facts from U.S. data on inventories that have proved to be quite a challenge to explain - sales that are less volatile than production, inventory investment that are procyclical and inventories-sales ratio which is countercyclical.

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Appendix A

Household's optimality conditions

Consider household's problem (1.2.2). Denoting by λ_1, λ_2 the Lagrange multipliers on first two constraints, the first order conditions and the envelope theorem conditions are

$$
\begin{aligned}
c: &\qquad 0 = U_c - \lambda_1 p - \lambda_2 \\
d: &\qquad 0 = U_d + \lambda_2 \psi^d \\
a': &\qquad 0 = -\lambda_1 + \beta \mathbb{E} W'_a \\
a: &\qquad W_a = \lambda_1 (1+R) \\
n: &\qquad W_n = U_n + \lambda_1 w + (1-\delta-\pi^u)\beta \mathbb{E} W'_n
\end{aligned}
$$

From the first order conditions for c and d we get for the value of the marginal unit of income

$$\lambda_1 = \frac{1}{p}\Big(U_c + \frac{U_d}{\psi^d}\Big)$$

and the envelope theorem yields the following expression for the marginal value of a worker employed under a contract at equilibrium wage $w(\mathbf{S})$

$$W_n = U_n + \Big(U_c + \frac{U_d}{\psi^d}\Big)\frac{w}{p} + (1-\delta-\pi^u)\beta \mathbb{E} W'_n \tag{A.1.1}$$

From the first order condition for a and envelope theorem for a'

$$\lambda_1 = \beta \mathbb{E}[\lambda'_1 (1+R')]$$

Plugging in for λ_1 yields the following Euler equation equalizing the cost of increasing saving in the form of share holdings by a marginal unit and the return from this marginal savings

$$\frac{1}{p}\Big(U_c + \frac{U_d}{\psi^d}\Big) = \beta \mathbb{E}\Big[(1+R')\frac{1}{p'}\Big(U'_c + \frac{U'_d}{(\psi^d)'}\Big)\Big] \tag{A.1.2}$$

The left hand side corresponds to the utility cost of extra unit of savings: the household could have instead purchased $\frac{1}{p}$ units of good which require utility cost $\frac{U_d}{\psi^d}$ per unit of good because of the goods market search friction, and enjoyed U_c extra utility per unit of good. The right hand side corresponds to the utility benefit of extra unit of savings: the $1+R$ monetary flow in the next period can be used to purchase extra consumption in the next period. It will be convenient to denote by $M(\mathbf{S})$ the expected discounted utility from marginal unit of share holdings

$$M = \beta\mathbb{E}\Big[(1+R')\frac{1}{p'}\Big(U_c' + \frac{U_d'}{(\psi^d)'}\Big)\Big] \tag{A.1.3}$$

The above intertemporal optimality conditions thus states that $\lambda_1 = M$.

Firm's optimality conditions

Since the household is representative adding the full set of Arrow securities would not affect the allocation, and we can use standard complete markets pricing approach to value the firm. Thus define the stochastic discount factor as

$$m(\mathbf{S},\mathbf{S}') = \beta\frac{p(\mathbf{S})}{p(\mathbf{S}')}\frac{U_c(\mathbf{S}') + \frac{U_d(\mathbf{S}')}{\psi^d(Q(\mathbf{S}'))}}{U_c(\mathbf{S}) + \frac{U_d(\mathbf{S})}{\psi^d(Q(\mathbf{S}))}} \tag{A.1.4}$$

where, with slight notational abuse $U_c(\mathbf{S}) = \frac{\partial}{\partial c}U(C(\mathbf{S}), D(\mathbf{S}), N(\mathbf{S}))$ and $U_d(\mathbf{S}) = \frac{\partial}{\partial d}U(C(\mathbf{S}), D(\mathbf{S}), N(\mathbf{S}))$.

Consider now firm's problem (1.2.3). After eliminating p, x, n' using the constraints the first order conditions and the envelope theorem condition are

$$\begin{aligned}
Q: &\qquad 0 = \left[\left(\frac{\psi^d_Q}{Q} - \frac{\psi^d}{Q^2}\right)\left(\frac{U_d - W^*_d}{\psi^d M} + \frac{U_c}{M}\right) - \frac{\psi^d}{Q}\frac{U_d - W^*_d}{(\psi^d)^2 M}\psi^d_Q\right]\frac{x}{X} \\
v: &\qquad 0 = -\frac{1}{X}\frac{\psi^d}{Q}p(\chi z f_l + \kappa_v) + \pi^v\mathbb{E}[m\Omega_n'] \\
n: &\qquad \Omega_n = -w + \frac{1}{X}\frac{\psi^d}{Q}pzf_l + (1-\delta)\mathbb{E}[m\Omega_n']
\end{aligned}$$

Using the first order condition for Q one can obtain that the equilibrium price in active market satisfies

$$p = \epsilon^{\psi^d}_Q\frac{U_c}{M} \tag{A.1.5}$$

where $\epsilon^{\psi^d}_Q = \frac{\partial\log\psi^d}{\partial\log Q}$. Without goods market search friction price of the good p in the market would be equal to the marginal rate of substitution between consumption and savings; with search friction the price is lower, since this helps the firm to attract more shoppers and increases the probability of selling the goods.

Next, applying the envelope theorem and using the first order condition for v together with (A.1.5) we can obtain the value of a marginal worker to a firm

$$\Omega_n = \left(zf_l + \frac{1-\delta}{\pi^v}(\chi z f_l + \kappa_v)\right)\frac{1}{X}\frac{\psi^d}{Q}p - w \tag{A.1.6}$$

Finally, combining (A.1.6) and the first order condition for v yields the job creation condition

$$\frac{1}{X}\frac{\psi^d}{Q}p(\chi z f_l + \kappa_v) = \pi^v \mathbb{E}\left[m\left(\left(z'f_l' + \frac{1-\delta}{(\pi^v)'}(\chi z' f_l' + \kappa_v')\right)\frac{1}{X'}\frac{(\psi^d)'}{Q'}p' - w'\right)\right] \tag{A.1.7}$$

Appendix B

Goods market

From (A.1.5) we have $M = \epsilon_Q^{\psi^d}\frac{U_c}{p}$ and so noting that the right hand side of equation (A.1.2) was defined in equation (A.1.3) to be M, we get

$$-U_d = (1-\epsilon_Q^{\psi^d})\psi^d U_c \tag{B.1.1}$$

Labor market

Under Nash bargaining wage w is determined as a solution to the following problem

$$w(\mathbf{S}) = \underset{\hat{w}}{\operatorname{argmax}}\, \hat{W}_n(\hat{w})^{\mu}\hat{\Omega}_n(\hat{w})^{1-\mu}$$

where $\hat{W}_n(\hat{w})$ and $\hat{\Omega}_n(\hat{w})$ are values, to household and firm, of a marginal worker employed under a contract with arbitrary wage $\hat{w}$ in the current period and equilibrium wage w thereafter, until the job is hit by the separation shock δ. I start by deriving these values.

The value function of a household with n members employed and earning equilibrium wage w, and ν members employed and earning arbitrary wage $\hat{w}$ in the current period and equilibrium wage w thereafter, until the they are hit by the separation shock δ is

$$
\begin{aligned}
&\tilde{W}(\hat{w}, \nu; \mathbf{S}) = \max_{c,d,a'} U(c, d, n+\nu, \zeta) + \beta \mathbb{E} W(s', n'; \mathbf{S}') \\
&\text{subject to} \\
&p(\mathbf{S})c + a' = (1 + R(\mathbf{S}))a + nw(\mathbf{S}) + \nu\hat{w} \\
&c = d\psi^d(Q(\mathbf{S}), X(\mathbf{S})) \\
&n' = (1-\delta)(n+\nu) + \pi^u(\theta(\mathbf{S}))(1-n-\nu) \\
&\mathbf{S}' = G(\mathbf{S})
\end{aligned}
$$

The value of a marginal worker earning wage $\hat{w}$ for this households can then be obtained as

$$
\hat{W}_n(\hat{w}) = \tilde{W}_\nu(\hat{w}, 0; \mathbf{S}) = \Big(U_c + \frac{U_d}{\psi^d}\Big)\frac{\hat{w} - w}{p} + W_n \tag{B.1.2}
$$

where the last part makes use of equation (A.1.1).

The value of a firm that employs n worker for equilibrium wage w, and ν workers for arbitrary wage $\hat{w}$ in the current period, and equilibrium wage w thereafter is

$$
\begin{aligned}
&\tilde{\Omega}(\hat{w}, \nu; \mathbf{S}) = \max_{v,p,Q,x} \left\{ \frac{x}{X(\mathbf{S})} \frac{\psi^d(Q, X(\mathbf{S}))}{Q} p - w(\mathbf{S})n - \hat{w}\nu + \mathbb{E}[m(\mathbf{S}, \mathbf{S}')\Omega(n'; \mathbf{S}')] \right\} \\
&\text{subject to} \\
&x = zf(n + \nu - \chi v) - \kappa(v) \\
&n' = (1-\delta)(n+\nu) + \pi^v(\theta(\mathbf{S}))v \\
&W_d^*(\mathbf{S}) = U_d(\mathbf{S}) + \psi^d(Q, X(\mathbf{S}))\big(U_c(\mathbf{S}) - pM(\mathbf{S})\big) \\
&\mathbf{S}' = G(\mathbf{S})
\end{aligned}
$$

Application of envelope theorem yields

$$
\tilde{\Omega}_\nu = -\hat{w} + \frac{1}{X}\frac{\psi^d}{Q} pzf_l + (1-\delta)\mathbb{E}[m\Omega_n']
$$

Notice that the first order conditions for Q, v are same as those in Appendix A, and thus we obtain for the value of a marginal worker that the firm employs and pays arbitrary wage $\hat{w}$ in the current period, and equilibrium wage w thereafter

$$
\hat{\Omega}_n(\hat{w}) = \tilde{\Omega}_\nu(\hat{w}, 0) = \left(zf_l + \frac{1-\delta}{\pi^v}(\chi zf_l + \kappa_v)\right)\frac{1}{X}\frac{\psi^d}{Q}p - \hat{w} = w - \hat{w} + \Omega_n \tag{B.1.3}
$$

The Nash bargaining problem is thus[1]

$$w(\mathbf{S}) = \underset{\hat{w}}{\operatorname{argmax}}\, \hat{W}_n(\hat{w})^{\mu}\hat{\Omega}_n(\hat{w})^{1-\mu}$$

with $\hat{W}_n(\hat{w}), \hat{\Omega}_n(\hat{w})$ given by (B.1.2) and (B.1.3). The first order condition yields a sharing rule

$$W_n = \frac{\mu}{1-\mu}\left(U_c + \frac{U_d}{\psi^d}\right)\frac{1}{p}\Omega_n \tag{B.1.4}$$

or $\frac{W_n}{p\lambda_1} = \mu\mathcal{S}$ where $\lambda_1 = \frac{1}{p}\big(U_c + \frac{U_d}{\psi^d}\big)$ is the marginal value of wealth for the household and $\mathcal{S} = \frac{\Omega_n}{p} + \frac{W_n}{p\lambda_1}$ is the total surplus of the match.

To derive the wage equation first plug W_n from the sharing rule (B.1.4) into (A.1.1), use stochastic discount factor (A.1.4), and the optimality conditions (A.1.7) and (A.1.6) which after a little bit of algebra yields the stochastic wage equation

$$\frac{w}{p} = \mu\frac{1}{X}\frac{\psi^d}{Q}(zf_l + \theta(\chi z f_l + \kappa_v)) - (1-\mu)\frac{U_n}{U_c + \frac{U_d}{\psi^d}} \tag{B.1.5}$$

Similarly to other search-matching models with Nash bargaining, wage is a weighted average of the value of marginal product of a worker enhanced by the vacancy cost savings, and the marginal rate of substitution between leisure and consumption.

Finally, to obtain a stochastic difference equation that characterizes the labor market plug for w' from (B.1.5) into job creation condition (A.1.7), and use stochastic discount factor (A.1.4), to get

$$\begin{aligned}&\frac{1}{\pi^v}(\chi z f_l + \kappa_v)\psi^x\left(U_c + \frac{U_d}{\psi^d}\right)\\ &= \beta\mathbb{E}\left[\left[(1-\mu)z' f_l' + \left(\frac{1-\delta}{(\pi^v)'} - \mu\theta'\right)(\chi z' f_l' + \kappa_v')\right](\psi^x)'\left(U_c' + \frac{U_d'}{(\psi^d)'}\right) + (1-\mu)U_n'\right]\end{aligned} \tag{B.1.6}$$

[1] For simplicity, this specification of the wage bargaining problem disregards the impact of losing a marginal worker on the bargaining position of firm with the remaining workers, see Stole and Zwiebel (1996).

Appendix C

Proof of Proposition 1

Proof. Consider the social planner's problem

$$\begin{aligned}
&\mathcal{W}(z,\zeta,N) = \max_{C,D,X,V} \left\{ U(C,D,N,\zeta) + \beta \mathbb{E}\mathcal{W}(z',\zeta',N') \right\} \\
&\text{subject to} \\
&C = m^G(D,X) \\
&X = zf(N-\chi V) - \kappa(V) \\
&N' = (1-\delta)N + m^L(1-N,V)
\end{aligned}$$

First, by combining the first order conditions for C and D we can obtain the intratemporal optimality condition equalizing the cost and the benefit of the marginal search effort

$$-U_D = m^G_D U_C \tag{C.1.1}$$

Next, using the first order condition for V and C we get for the marginal value of an employed worker

$$\mathcal{W}_N = U_N + \Big(zf_L + \frac{1-\delta-m^L_U}{m^L_V}(\chi z f_L + \kappa_V) \Big) m^G_X U_C$$

and by shifting this one period forward and plugging back into the first order condition for V we get the following intertemporal optimality condition

$$\begin{aligned}
&m^G_X(\chi z f_L + \kappa_V) U_C \\
&\quad = m^L_V \beta \mathbb{E}\Big[\Big[z' f'_L + \frac{1-\delta-(m^L_U)'}{(m^L_V)'}(\chi z' f'_L + \kappa'_V) \Big] (m^G_X)' U'_C + U'_N \Big]
\end{aligned} \tag{C.1.2}$$

To summarize, efficient allocation is characterized by (C.1.1), (C.1.2) and constraints

$$\begin{aligned}
C &= m^G(D,X) \\
X &= zf(N-\chi V) - \kappa(V) \\
N' &= (1-\delta)N + m^L(1-N,V)
\end{aligned}$$

The equilibrium allocation on the other hand satisfies these three constraints and (B.1.1), (B.1.6). It is easy to verify that since $\epsilon_Q^{\psi^d} = \frac{\partial \log \psi^d}{\partial \log Q}$, $\epsilon_\theta^{\pi^v} = \frac{d \log \pi^v}{d \log \theta}$ it holds that

$$m_D^G = (1 - \epsilon_Q^{\psi^d})\psi^d \qquad m_X^G = \frac{1}{X}\psi_Q^d \qquad m_U^L = -\epsilon_\theta^{\pi^v}\pi^u \qquad m_V^L = (1 + \epsilon_\theta^{\pi^v})\pi^v$$

so that (C.1.1) can be rewritten as

$$-U_D = (1 - \epsilon_Q^{\psi^d})\psi^d U_C$$

and thus (C.1.2) becomes

$$\begin{aligned}
&\frac{1}{\pi^v}(\chi z f_L + \kappa_V)\psi^x\left(U_C + \frac{U_D}{\psi^d}\right) \\
&= \beta\mathbb{E}\Bigg[\Bigg[(1 + \epsilon_\theta^{\pi^v})z' f_L' + \frac{1 + \epsilon_\theta^{\pi^v}}{1 + (\epsilon_\theta^{\pi^v})'}\left(\frac{1-\delta}{(\pi^v)'} + (\epsilon_\theta^{\pi^v})'\theta'\right)(\chi z' f_L' + \kappa_V')\Bigg] \\
&\qquad \times (\psi^x)'\left(U_C' + \frac{U_D'}{(\psi^d)'}\right) + (1 + \epsilon_\theta^{\pi^v})U_N'\Bigg]
\end{aligned}$$

Clearly, if $\mu = \frac{\partial \log m^L}{\partial \log U} = -\epsilon_\theta^{\pi^v}$ these are exactly the same conditions as (B.1.1) and (B.1.6), and so the conditions for efficient allocation coincide with conditions for equilibrium allocation. $\square$

The following lemma is used throughout in subsequent proofs.

Lemma 2. *Suppose that $f(x, y)$ is homogeneous of degree 1 and has elasticity of substitution σ_{xy}. Then $\sigma_{xy} = \frac{f_x f_y}{f_{xy} f}$.*

Proof. Since f has elasticity of substitution σ_{xy}

$$\sigma_{xy} = \frac{\frac{1}{x f_x} + \frac{1}{y f_y}}{-\frac{f_{xx}}{(f_x)^2} + \frac{2 f_{xy}}{f_x f_y} - \frac{f_{yy}}{(f_y)^2}}$$

thus

$$f_y y + f_x x = \sigma_{xy}\left[-xy\left[f_{xx}\frac{f_y}{f_x} + f_{yy}\frac{f_x}{f_y}\right] + 2 f_{xy} xy\right]$$

Using the fact that f is HOD 1 we have

$$f = -\sigma_{xy}\left[xy\left[f_{xx}\frac{f_y}{f_x} + f_{yy}\frac{f_x}{f_y}\right] + f_{xx}x^2 + f_{yy}y^2\right]$$
$$= -\sigma_{xy}\left[f_{xx}\frac{x}{f_x}(f_x x + f_y y) + f_{yy}\frac{y}{f_y}(f_y y + f_x x)\right] = -\sigma_{xy}\left[\frac{f_{xx}x}{f_x} + \frac{f_{yy}y}{f_y}\right]f$$

Now use the fact that f_x and f_y are HOD 0 and simplify further to get

$$1 = -\sigma_{xy}\left[\frac{-f_{xy}y}{f_x} + \frac{-f_{xy}x}{f_y}\right] = \sigma_{xy}\frac{f_{xy}f}{f_x f_y}$$

□

Proof of Proposition 2

Proof.

a. First, using $y = \frac{\psi^d}{QN}$ and $\psi^d_Q = \epsilon^{\psi^d}_Q\frac{\psi^d}{Q}$ we have $\psi^d_Q = \epsilon^{\psi^d}_Q Ny$, and since under Assumption 2 $\kappa = z\bar{\kappa}$ and $X = z(f - \bar{\kappa})$, the goods and labor market equilibrium conditions (1.3.1) and (1.3.2) become

$$-U_d D = (1 - \epsilon^{\psi^d}_Q)U_c C \tag{C.1.3}$$

$$\frac{1}{\pi^v}\frac{\chi f_l + \bar{\kappa}_v}{f - \bar{\kappa}}\epsilon^{\psi^d}_Q NyU_c = \beta\mathbb{E}\left[\left((1-\mu)f'_l + \left(\frac{1-\delta}{(\pi^v)'} - \mu\theta'\right)(\chi f'_l + \kappa'_v)\right)\frac{1}{f' - \bar{\kappa}'}(\epsilon^{\psi^d}_Q)'N'y'U'_c + (1-\mu)U'_n\right] \tag{C.1.4}$$

Consider now any arbitrary history of shocks (z^t, ζ^t) and resulting history of measured average labor productivity, market tightness and employment $(y^t, \theta^t, Q^t, N^t)$ which satisfy equilibrium conditions (C.1.3), (C.1.4) and the law of motion for labor (1.3.3). Now let $\tilde{z}_t = 1$; because m^G is strictly increasing in both its arguments we can find a unique $\tilde{Q}_t$ such that

$$\frac{\psi^d(\tilde{Q}_t, \frac{\tilde{z}_t}{z_t}X_t)}{\tilde{Q}_t} = \frac{\psi^d(Q_t, X_t)}{Q_t} \tag{C.1.5}$$

and set $\tilde{D}_t = 1/\tilde{Q}_t$ and $\tilde{X}_t = \tilde{z}_t f\big(N_t - \chi\theta_t(1-N_t)\big) - \tilde{z}_t\bar{\kappa}\big(\theta_t(1-N_t)\big)$. This guarantees that

$$C_t = \psi^d(\tilde{Q}_t\tilde{X}_t)/\tilde{Q}_t$$
$$y_t = \frac{\psi^d(\tilde{Q}_t, \tilde{X}_t)/\tilde{Q}_t}{N_t}$$

hold. Let $\tilde{\zeta}_{dt}$ be such that (C.1.3) holds with $(Q_t, D_t, X_t, \zeta_{dt})$ replaced by $(\tilde{Q}_t, \tilde{D}_t, \tilde{X}_t, \zeta_{dt})$, that is

$$-U_d\big(C_t, \tilde{D}_t, N_t, (\zeta_{ct}, \tilde{\zeta}_{dt}, \zeta_{nt})\big)\tilde{D}_t = (1 - \epsilon_Q^{\psi^d}(\tilde{Q}_t, \tilde{X}_t))U_c\big(C_t, \tilde{D}_t, N_t, (\zeta_{ct}, \tilde{\zeta}_{dt}, \zeta_{nt})\big)C_t$$

The last equilibrium condition that is left to be verified is (C.1.4). Since by Assumption 1A preferences are additively separable the optimal allocation for household satisfies $c_e = c_u = c$ and $d_e = d_u = d$ and so

$$U(c, d, n, \zeta) = \zeta_c u(c) - \zeta_d g(d) - \zeta_n n$$

which implies that

$$U_c\big(C_t, \tilde{D}_t, N_t, (\zeta_{ct}, \tilde{\zeta}_{dt}, \zeta_{nt})\big) = U_c\big(C_t, D_t, N_t, (\zeta_{ct}, \zeta_{dt}, \zeta_{nt})\big) \tag{C.1.6}$$

$$U_n\big(C_t, \tilde{D}_t, N_t, (\zeta_{ct}, \tilde{\zeta}_{dt}, \zeta_{nt})\big) = U_n\big(C_t, D_t, N_t, (\zeta_{ct}, \zeta_{dt}, \zeta_{nt})\big) \tag{C.1.7}$$

and thus (C.1.4) will hold for $((\tilde{z}^t, \tilde{\zeta}^t), (y^t, \theta^t, \tilde{Q}^t, N^t))$ as long as

$$\epsilon_Q^{\psi^d}(\tilde{Q}_t, \tilde{X}_t) = \epsilon_Q^{\psi^d}(Q_t, X_t) \tag{C.1.8}$$

It now only remains to be shown that this condition can only be satisfied if $\epsilon_Q^{\psi^d}(Q, X) \equiv const$. Since by construction

$$\epsilon_Q^{\psi^d} = m_Q^G(1, QX)\frac{Q}{m^G(1, QX)} = m_{(TX)}^G(D, TX)\frac{TX}{m^G(D, TX)}$$

and $\tilde{X}_t = \frac{\tilde{z}_t}{z_t}X_t$, condition (C.1.8) is equivalent to

$$m_{(TX)}^G\Big(\tilde{D}_t, T\frac{\tilde{z}_t}{z_t}X_t\Big)\frac{T\frac{\tilde{z}_t}{z_t}X_t}{m^G\Big(\tilde{D}_t, T\frac{\tilde{z}_t}{z_t}X_t\Big)} = m_{(TX)}^G(D_t, TX_t)\frac{TX_t}{m^G(D_t, TX_t)}$$

and since m^G is homogeneous of degree 1 and $m_{(TX)}^G$ homogeneous of degree 0, this implies $\tilde{D}_t = \frac{\tilde{z}_t}{z_t}D_t$. But then (C.1.5) would imply

$$\frac{\tilde{z}_t}{z_t}m^G(D_t, TX_t) = m^G(\tilde{D}_t, T\tilde{X}_t) = \frac{\psi^d(\tilde{Q}_t, \frac{\tilde{z}_t}{z_t}X_t)}{\tilde{Q}_t} = \frac{\psi^d(Q_t, X_t)}{Q_t} = m^G(D_t, TX_t)$$

or $\tilde{z}_t = z_t$ which is a contradiction. Thus (C.1.8) can only be satisfied if matching function m^G is actually such that $\epsilon_Q^{\psi^d}(Q, X) \equiv const$.

b. Since the wage is given by (B.1.5), using (B.1.1), (C.1.5) and (C.1.6)-(C.1.7) we obtain

$$\frac{w}{p} = \mu \frac{1}{\tilde{X}} \frac{\tilde{\psi}^d}{\tilde{Q}} (\tilde{z} f_l + \theta(\chi \tilde{z} f_l + \tilde{z}\bar{\kappa}_v)) - (1-\mu) \frac{\tilde{U}_n}{\epsilon_Q^{\psi^d} \tilde{U}_c}$$

Thus the observed histories of real wages under history of shocks (z^t, ζ^t) and under the alternative history $(\tilde{z}^t, \tilde{\zeta}^t)$ are identical only if $\epsilon_Q^{\psi^d}(\tilde{Q}_t, \tilde{X}_t) = \epsilon_Q^{\psi^d}(Q_t, X_t)$ for all t, which as shown in part a. only holds if $\epsilon_Q^{\psi^d}(Q, X)$ is actually constant.

□

Proof of Proposition 3

Proof. Under Assumption 1A with $u(c) = \log c$, since preferences are additively separable, optimal allocation for household satisfies $c_e = c_u = c$, $d_e = d_u = d$ and so

$$U(C, D, N, \zeta) = \zeta_c \log C - \zeta_d g(D) - \zeta_n N$$

Then

a. Since $m_D^G = (1 - \epsilon_Q^{\psi^d})\psi^d$ the goods market equation (1.3.1) can be written as

$$0 = m_D^G(D, TX) U_C\big(m^G(D, TX), D, N, \zeta\big) + U_D\big(m^G(D, TX), D, N, \zeta\big)$$

where $X = zf(N - \chi V) - \kappa(V)$. Then, under Assumption 1A with $u(c) = \log c$, this simplifies to

$$\zeta_d g_D D = \zeta_c \epsilon_D^{m^G} \tag{C.1.9}$$

Because

$$\frac{\partial \epsilon_D^{m^G}}{\partial (TX)} = \frac{m_{D,(TX)}^G D}{m^G}\Big(1 - \frac{m_D^G m_{TX}^G}{m_{D,(TX)}^G m^G}\Big) = -\frac{1}{TX}\frac{1}{\sigma}\epsilon_{(TX)}^{m^G}\epsilon_D^{m^G}(1-\sigma)$$

if the matching function m^G has elasticity of substitution $\sigma = 1$ then (C.1.9) does not depend on X, thus the search effort D and consequently also goods market tightness Q in equilibrium does not react to changes in productivity z.

b. Under Assumption 1A with $u(c) = \log c$ labor market equation (1.3.2) becomes

$$\frac{1}{X}\epsilon_Q^{\psi^d}\frac{K}{\pi^v}\zeta_c = \beta \mathbb{E}\Big[\Big[(1-\mu)z' f_l' + \Big(\frac{1-\delta}{(\pi^v)'} - \mu\theta'\Big)K'\Big]\frac{1}{X'}(\epsilon_Q^{\psi^d})'\zeta_c' + (1-\mu)U_n'\Big] \tag{C.1.10}$$

where $K = \chi z f_L(N - \chi\theta(1-N)) + \kappa_V(\theta(1-N))$ and $X = zf(N - \chi\theta(1-N)) - \kappa(\theta(1-N))$. Moreover, ζ_d does not enter the equation above explicitly, so any changes in ζ_d can only affect labor market indirectly through changes in Q and $\epsilon_Q^{\psi^d}$. Since

$$\epsilon_Q^{\psi^d} = \frac{\partial m^G(1, QX)}{\partial Q}\frac{Q}{m^G(1, QX)} = \frac{m^G_{(TX)}(1, QX)XQ}{m^G(1, QX)} = \frac{m^G_{(TX)}(D, TX)TX}{m^G(D, TX)} = \epsilon^{m^G}_{(TX)}$$

and

$$\frac{\partial \epsilon^{m^G}_{(TX)}}{\partial D} = \frac{m^G_{D,(TX)}TX}{m^G}\left(1 - \frac{m^G_D m^G_{(TX)}}{m^G_{D,(TX)}m^G}\right) = -\frac{1}{D}\frac{1}{\sigma}\epsilon^{m^G}_{(TX)}\epsilon^{m^G}_D(1-\sigma)$$

if the matching function m^G has elasticity of substitution $\sigma = 1$ then Q does not enter equation (C.1.10), and thus the labor market tightness θ is unaffected by shocks to the disutility from search ζ_d.

c. If Assumption 2 also holds in addition to Assumption 1A, then $K = z\bar{K}$ and $X = z\bar{X}$ where $\bar{K} = \chi f_L(N - \chi\theta(1-N)) + \bar{\kappa}_V(\theta(1-N))$ and $\bar{X} = f(N - \chi\theta(1-N)) - \bar{\kappa}(\theta(1-N))$; thus the labor market equation (C.1.10) simplifies even further and becomes

$$\frac{1}{\bar{X}}\epsilon_Q^{\psi^d}\frac{\bar{K}}{\pi^v}\zeta_c = \beta\mathbb{E}\left[\left[(1-\mu)f_l' + \left(\frac{1-\delta}{(\pi^v)'} - \mu\theta'\right)\bar{K}'\right]\frac{1}{\bar{X}'}(\epsilon_Q^{\psi^d})'\zeta_c' + (1-\mu)U_n'\right]$$

Since z does not enter this equation explicitly, if the matching function m^G has elasticity of substitution $\sigma = 1$ so that using results from parts a. and b. neither Q nor $\epsilon_Q^{\psi^d}$ react to changes in productivity z, then labor market tightness θ is also unaffected by shocks that change the productivity z. □

Proof of Lemma 1

Proof. First, using $y = \frac{\psi^d}{QN}$ and $\psi^d_Q = \epsilon_Q^{\psi^d}\frac{\psi^d}{Q}$ we have $\psi^d_Q = \epsilon_Q^{\psi^d}Ny$, and since under Assumption 2 $\kappa = z\bar{\kappa}$ and $X = z(f - \bar{\kappa})$, it is straightforward to show that the goods and labor market equilibrium conditions (1.3.1) and (1.3.2) become

$$0 = (1 - \epsilon_Q^{\psi^d})U_C C + U_D D$$
$$0 = \left((1-\mu)f_L - \left(\mu\theta + \frac{1-\beta(1-\delta)}{\pi^v}\right)(\chi f_L + \bar{\kappa}_V)\right)\frac{N}{f - \bar{\kappa}}y + (1-\mu)\frac{U_N}{\epsilon_Q^{\psi^d}U_C}$$

and similarly real wage (1.3.5) can be rewritten as

$$\frac{w}{p} = \mu(f_L + \theta(\chi f_L + \bar{\kappa}_V))\frac{N}{f - \bar{\kappa}}y - (1-\mu)\frac{U_N}{\epsilon_Q^{\psi^d} U_C}$$

Note that $m_D^G = (1 - \epsilon_Q^{\psi^d})\psi^d$ and $\frac{\psi_Q^d}{X} = m_{(TX)}^G$, thus in the steady (1.3.1) and (1.3.2) can be also rewritten as

$$\begin{aligned} 0 &= m_D^G U_C + U_D \\ 0 &= \Lambda z m_{(TX)}^G U_C + (1-\mu)U_N \end{aligned}$$

where $\Lambda = (1-\mu)f_L - \big(\mu\theta + \frac{1-\beta(1-\delta)}{\beta\pi^v}\big)(\chi f_L + \bar{\kappa}_V)$.

If we denote $\mathbf{Q} = (Q, \theta)'$ and $\mathbf{x} = (z, \zeta)'$ we can thus define function $\mathbf{G}$ in order to write the above two conditions as $\mathbf{0} = \mathbf{G}(\mathbf{Q}, \mathbf{x})$. Applying Implicit Function Theorem, we then obtain $\mathbf{Q} = \mathbf{F}(\mathbf{x})$ and $\mathbf{G_Q} d\mathbf{Q} = -\mathbf{G_x} d\mathbf{x}$, afterwards using Cramer's rule we get

$$\epsilon_z^{\theta^{GLS}} = -\frac{G_Q^1 G_z^2 - G_Q^2 G_z^1}{G_Q^1 G_\theta^2 - G_Q^2 G_\theta^1}\frac{z}{\theta}$$

and then, since $G_Q^2 dQ + G_\theta^2 d\theta + G_z^2 dz = 0$

$$\epsilon_z^Q = -\frac{G_z^2 z}{G_Q^2 Q} - \frac{G_\theta^2 \theta}{G_Q^2 Q}\epsilon_z^{\theta^{GLS}}$$

It is straightforward to verify that

$$\begin{aligned} G_z^2 &= -(1-\mu)\left(1 - \frac{\epsilon_D^{m^G}}{\sigma} + (1-\epsilon_D^{m^G})\epsilon_C^{MRS_{CN}}\right)\frac{U_N}{z} \\ G_\theta^2 &= -(1-\mu)\left(\epsilon_\theta^\Lambda - \frac{\epsilon_D^{m^G}}{\sigma}\epsilon_\theta^X + \epsilon_C^{MRS_{CN}}\epsilon_{TX}^{m^G}\epsilon_\theta^X + \epsilon_N^{MRS_{CN}}\epsilon_\theta^N\right)\frac{U_N}{\theta} \\ G_Q^2 &= (1-\mu)\left(\frac{\epsilon_D^{m^G}}{\sigma}\big(1 + \sigma\epsilon_C^{MRS_{CN}}\big) + \epsilon_D^{MRS_{CN}}\right)\frac{U_N}{Q} \end{aligned}$$

so that

$$\epsilon_z^Q = \frac{1 - \frac{\epsilon_D^{m^G}}{\sigma} + (1-\epsilon_D^{m^G})\epsilon_C^{MRS_{CN}} - \Big(-\epsilon_\theta^\Lambda + \frac{\epsilon_D^{m^G}}{\sigma}\epsilon_\theta^X - \epsilon_C^{MRS_{CN}}\epsilon_{TX}^{m^G}\epsilon_\theta^X - \epsilon_N^{MRS_{CN}}\epsilon_\theta^N\Big)\epsilon_z^{\theta^{GLS}}}{\frac{\epsilon_D^{m^G}}{\sigma}\big(1 + \sigma\epsilon_C^{MRS_{CN}}\big) + \epsilon_D^{MRS_{CN}}} \tag{C.1.11}$$

Note that in the model with labor search only, the steady state θ satisfies

$$0 = G(\theta, z) = \Lambda z U_C + (1-\mu)U_N$$

and since

$$G_z = -(1-\mu)\big(1+\epsilon_C^{MRS_{CN}}\big)\frac{U_N}{z}$$
$$G_\theta = -(1-\mu)\Big(\epsilon_\theta^{\Lambda} + \epsilon_C^{MRS_{CN}}\epsilon_\theta^{X} + \epsilon_N^{MRS_{CN}}\epsilon_\theta^{N}\Big)\frac{U_N}{\theta}$$

we get

$$\epsilon_z^{\theta^{LS}} = -\frac{G_z}{G_\theta} = \frac{1+\epsilon_C^{MRS_{CN}}}{-\epsilon_\theta^{\Lambda} - \epsilon_C^{MRS_{CN}}\epsilon_\theta^{X} - \epsilon_N^{MRS_{CN}}\epsilon_\theta^{N}} \tag{C.1.12}$$

Because steady state θ and N are calibrated to be the same in the model with labor search and the model with goods and labor search, assuming that preferences and technology are such that the steady state elasticities $\epsilon_C^{U_C}$, $\epsilon_N^{U_C}$, $\epsilon_C^{U_N}$, $\epsilon_N^{U_N}$, ϵ_L^{f}, $\epsilon_L^{f_L}$ are same in both models, and also $\mu^{LS} = \mu^{GLS}$ we can use (C.1.12) to substitute for $\epsilon_\theta^{\Lambda}$ into (C.1.11) to obtain after some rearrangements

$$\frac{1}{\epsilon_z^{\theta^{LS}}} = \left(1 - \frac{1}{\sigma}\frac{1+\sigma\epsilon_C^{MRS_{CN}}}{1+\epsilon_C^{MRS_{CN}}}\epsilon_D^{m^G}(1+\epsilon_z^{Q}+\epsilon_\theta^{X}\epsilon_z^{\theta^{GLS}}) - \frac{\epsilon_D^{MRS_{CN}}}{1+\epsilon_C^{MRS_{CN}}}\epsilon_z^{Q}\right)\frac{1}{\epsilon_z^{\theta^{GLS}}} \tag{C.1.13}$$

Since the model with labor search only measured productivity is given by $y = \frac{X}{N}$ thus

$$\epsilon_z^{y^{LS}} = 1 + \big(\epsilon_\theta^{X} - \epsilon_\theta^{N}\big)\epsilon_z^{\theta^{LS}} \tag{C.1.14}$$

Similarly, since in the model with goods and labor search $y = \frac{m^G}{N}$ we have

$$\epsilon_z^{y^{GLS}} = 1 + \big(\epsilon_\theta^{X} - \epsilon_\theta^{N}\big)\epsilon_z^{\theta^{GLS}} - \epsilon_D^{m^G}\big(1+\epsilon_z^{Q}+\epsilon_\theta^{X}\epsilon_z^{\theta^{GLS}}\big) \tag{C.1.15}$$

Because π^u, δ and the steady state θ and N are same in both models, we can combine (C.1.14) and (C.1.15) to obtain

$$\frac{\epsilon_z^{y^{GLS}}}{\epsilon_z^{\theta^{GLS}}} = \frac{\epsilon_z^{y^{LS}}}{\epsilon_z^{\theta^{LS}}} + \Big(1 - \epsilon_D^{m^G}(1+\epsilon_z^{Q}+\epsilon_\theta^{X}\epsilon_z^{\theta^{GLS}})\Big)\frac{1}{\epsilon_z^{\theta^{GLS}}} - \frac{1}{\epsilon_z^{\theta^{LS}}}$$

Finally, use (C.1.13) to substitute for $\frac{1}{\epsilon_z^{\theta^{LS}}}$ and get

$$\frac{1}{\epsilon_y^{\theta^{GLS}}} = \frac{1}{\epsilon_y^{\theta^{LS}}} + \frac{1}{1+\epsilon_C^{MRS_{CN}}}\left(\frac{1-\sigma}{\sigma}\epsilon_D^{m^G}\epsilon_z^{QX} + \epsilon_D^{MRS_{CN}}\epsilon_z^{Q}\right)\frac{1}{\epsilon_z^{\theta^{GLS}}}$$

$\square$

Proof of Proposition 4

Proof. First, note that $\epsilon_z^{QX} = \epsilon_z^Q + 1 + \epsilon_\theta^X \epsilon_z^\theta$ and that given $\mathbf{G}$ as defined in the proof of Lemma 1

$$\epsilon_z^Q = -\frac{G_z^1 z}{G_Q^1 Q} - \frac{G_\theta^1 \theta}{G_Q^1 Q}\epsilon_z^{\theta^{GLS}}$$

Since

$$G_z^1 = -\left(1+\sigma\epsilon_C^{MRS_{CD}}\right)\frac{1-\epsilon_D^{m^G}}{\sigma}\frac{U_D}{z}$$
$$G_\theta^1 = -\left(\left(1+\sigma\epsilon_C^{MRS_{CD}}\right)\frac{1-\epsilon_D^{m^G}}{\sigma}\epsilon_\theta^X + \epsilon_N^{MRS_{CD}}\epsilon_\theta^N\right)\frac{U_D}{\theta}$$
$$G_Q^1 = -\left(\frac{1-\epsilon_D^{m^G}}{\sigma} - \epsilon_D^{m^G}\epsilon_C^{MRS_{CD}} - \epsilon_D^{MRS_{CD}}\right)\frac{U_D}{Q}$$

we have

$$\epsilon_z^Q = \frac{-\frac{1-\epsilon_D^{m^G}}{\sigma}\left(1+\sigma\epsilon_C^{MRS_{CD}}\right) - \left(\frac{1-\epsilon_D^{m^G}}{\sigma}\left(1+\sigma\epsilon_C^{MRS_{CD}}\right)\epsilon_\theta^X + \epsilon_N^{MRS_{CD}}\epsilon_\theta^N\right)\epsilon_z^{\theta^{GLS}}}{\frac{1-\epsilon_D^{m^G}}{\sigma} - \epsilon_D^{m^G}\epsilon_C^{MRS_{CD}} - \epsilon_D^{MRS_{CD}}}$$

and

$$\epsilon_z^{QX} = -\frac{(\epsilon_C^{MRS_{CD}} + \epsilon_D^{MRS_{CD}})(1+\epsilon_\theta^X\epsilon_z^{\theta^{GLS}}) + \epsilon_N^{MRS_{CD}}\epsilon_\theta^N\epsilon_z^{\theta^{GLS}}}{\frac{1-\epsilon_D^{m^G}}{\sigma} - \epsilon_D^{m^G}\epsilon_C^{MRS_{CD}} - \epsilon_D^{MRS_{CD}}}$$

Under Assumptions 1A condition (1.3.9) then yields

$$\frac{1}{\epsilon_y^{\theta^{GLS}}} = \frac{1}{\epsilon_y^{\theta^{LS}}} + \frac{1}{1+\epsilon_C^{U_C}}\frac{1-\sigma}{\sigma}\epsilon_D^{m^G}\frac{(-\epsilon_C^{U_C}+\epsilon_D^{U_D})(1+\epsilon_\theta^X\epsilon_z^{\theta^{GLS}})}{\frac{1-\epsilon_D^{m^G}}{\sigma} - \epsilon_D^{m^G}\epsilon_C^{U_C} + \epsilon_D^{U_D}}\frac{1}{\epsilon_z^{\theta^{GLS}}}$$

Therefore if $\sigma > 1$ we have $\epsilon_y^{\theta^{GLS}} > \epsilon_y^{\theta^{LS}}$.
Under Assumptions 1B condition (1.3.9) then yields

$$\frac{1}{\epsilon_y^{\theta^{GLS}}} = \frac{1}{\epsilon_y^{\theta^{LS}}} + \frac{1}{1+\epsilon_C^{U_C}}\left(\frac{1-\sigma}{\sigma}\frac{\epsilon_D^{m^G}\epsilon_D^{g_D}(1+\epsilon_\theta^X\epsilon_z^{\theta^{GLS}})}{\frac{1-\epsilon_D^{m^G}}{\sigma} - \epsilon_D^{m^G}\epsilon_C^{U_C} + \epsilon_D^{U_D}} + \epsilon_D^{U_C}\epsilon_z^Q\right)\frac{1}{\epsilon_z^{\theta^{GLS}}}$$

Therefore $\epsilon_y^{\theta^{GLS}} > \epsilon_y^{\theta^{LS}}$ as long as u has relative risk aversion coefficient $-\epsilon_C^{U_C} \le 1$ and $\sigma \ge 1$; or alternatively $-\epsilon_C^{U_C} \le 1$ and $\sigma < 1$ and $\epsilon_D^{g_D}$ is sufficiently small. $\square$

Proof of Proposition 5

Proof. Since in the model with goods and labor search $y = \frac{m^G}{N}$ we have

$$\epsilon^y_{\zeta_d} = \left(\epsilon^X_\theta - \epsilon^N_\theta\right)\epsilon^{\theta^{GLS}}_{\zeta_d} - \epsilon^{m^G}_D\left(\epsilon^Q_{\zeta_d} + \epsilon^X_\theta \epsilon^{\theta^{GLS}}_{\zeta_d}\right) \tag{C.1.16}$$

and thus for $\frac{1}{\epsilon^{\theta^{GLS}}_y} = \frac{\epsilon^y_{\zeta_d}}{\epsilon^{\theta^{GLS}}_{\zeta_d}}$ we obtain

$$\frac{1}{\epsilon^{\theta^{GLS}}_y} = -\epsilon^{m^G}_D \frac{\epsilon^Q_{\zeta_d}}{\epsilon^{\theta^{GLS}}_{\zeta_d}} + (1 - \epsilon^{m^G}_D)\epsilon^X_\theta - \epsilon^N_\theta$$

Given function $\mathbf{G}$ as defined in the proof of Lemma 1 we get using Cramer's rule

$$\frac{\epsilon^Q_{\zeta_d}}{\epsilon^{\theta^{GLS}}_{\zeta_d}} = \frac{-\frac{G^1_{\zeta_d}G^2_\theta - G^2_{\zeta_d}G^1_\theta}{G^1_Q G^2_\theta - G^2_Q G^1_\theta}\frac{\zeta_d}{Q}}{-\frac{G^1_Q G^2_{\zeta_d} - G^1_{\zeta_d}G^2_Q}{G^1_Q G^2_\theta - G^1_\theta G^2_Q}\frac{\zeta_d}{\theta}} = -\frac{G^1_{\zeta_d}G^2_\theta - G^2_{\zeta_d}G^1_\theta}{G^1_{\zeta_d}G^2_Q - G^1_Q G^2_{\zeta_d}}\frac{\theta}{Q}$$

and with additively separable preferences from Assumptions 1A since $G^2_{\zeta_d} = 0$

$$\frac{\epsilon^Q_{\zeta_d}}{\epsilon^{\theta^{GLS}}_{\zeta_d}} = \frac{\epsilon^\Lambda_\theta - \frac{\epsilon^{m^G}_D}{\sigma}\epsilon^X_\theta + \epsilon^{m^G}_{(TX)}\epsilon^{U_C}_C\epsilon^X_\theta - \epsilon^{U_N}_N\epsilon^N_\theta}{\frac{\epsilon^{m^G}_D}{\sigma}(1 + \sigma\epsilon^{U_C}_C)}$$

Plugging this back yields

$$\frac{1}{\epsilon^{\theta^{GLS}}_y} = \frac{-\epsilon^\Lambda_\theta - \epsilon^{U_C}_C\epsilon^X_\theta + \epsilon^{U_N}_N\epsilon^N_\theta}{\frac{1}{\sigma} + \epsilon^{U_C}_C} + \epsilon^X_\theta - \epsilon^N_\theta \tag{C.1.17}$$

In the model with labor search only with a shock to z we have from (C.1.12) and (C.1.14) under Assumption 1A that

$$\frac{1}{\epsilon^{\theta^{LS}}_y} = \frac{-\epsilon^\Lambda_\theta - \epsilon^{U_C}_C\epsilon^X_\theta + \epsilon^{U_N}_N\epsilon^N_\theta}{1 + \epsilon^{U_C}_C} + \epsilon^X_\theta - \epsilon^N_\theta \tag{C.1.18}$$

comparing (C.1.17) and (C.1.18) the result follows immediately. □

Appendix D

Data

Seasonally adjusted average output per worker in nonfarm business sector y, and seasonally adjusted output in the nonfarm business sector Y are both constructed by the BLS using National Income and Product Accounts and Current Employment Survey, they are series PRS85006163 and PRS85006043 respectively.

Seasonally adjusted composite help wanted index V is constructed following the approach in Barnichon (2010) and combines help-wanted advertising index and the online help-wanted index constructed by Conference Board. Seasonally adjusted unemployment U is constructed by BLS from Current Population Survey, it's the series LNS13000000. Both V and U are quarterly averages of monthly series.

In simulations, quarterly average labor productivity y_t is calculated as quarterly output Y_t divided by the quarter's employment N_t, with quarterly output given by the sum of weekly output, and quarterly employment given by the average employment in the three months of the quarter. Since for each month employment is measured by the BLS in the second week quarterly productivity is calculated as

$$y_t = \frac{\sum_{i=1}^{12} Y^W_{12t-i+1}}{\frac{1}{3}(N^W_{12t-2} + N^W_{12t-6} + N^W_{12t-10})}$$

The ratio of inventories to sales ι is constructed using data for real nonfarm inventories and real final sales of domestic business from BEA, from Table 5.8.6 of the National Income and Product Accounts.

Log-linearized model

Let $\hat{x} = \log(x/\bar{x})$ denote the percentage deviation of variable x from its steady state $\bar{x}$. The log-linearized system of equations for $(\theta, Q, \Omega_i^r, I, N, y, \iota)$ is

$$
\begin{aligned}
&\frac{1}{\bar{\pi}^v}\Big(- \hat{\pi}^v + \hat{z} + \hat{f}_l + \hat{\Omega}_i^r + \bar{\epsilon}_Q^{\psi^d} + \hat{U}_c\Big) \\
&= \beta\mathbb{E}\Bigg[- \mu\bar{\theta}\hat{\theta}' - \frac{1-\delta}{\bar{\pi}^v}(\hat{\pi}^v)' + \Big(1 - \mu - \mu\bar{\theta} + \frac{1-\delta}{\bar{\pi}^v}\Big)\Big(\hat{z}' + \hat{f}_l' + (\hat{\Omega}_i^R)' + (\hat{\epsilon}_Q^{\psi^d})' + \hat{U}_c'\Big) \\
&\qquad - \Big(1 - \mu - \mu\bar{\theta} - \frac{1-\beta(1-\delta)}{\beta\bar{\pi}^v}\Big)\frac{1}{b\bar{\epsilon}_Q^{\psi^d}\bar{U}_c - \bar{U}_n}\big(b\bar{\epsilon}_Q^{\psi^d}\bar{U}_c(\hat{\epsilon}_Q^{\psi^d} + \hat{U}_c') - \bar{U}_n\hat{U}_n'\big)\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
\hat{U}_d &= \hat{\psi}^d - \frac{\bar{\epsilon}_Q^{\psi^d}}{1 - \bar{\epsilon}_Q^{\psi^d}}\hat{\epsilon}_Q^{\psi^d} + \frac{1}{\bar{U}_c - (1-\delta_i)\beta\Big(\bar{U}_c + \frac{\bar{U}_d}{\bar{\psi}^d}\Big)\bar{\Omega}_i^r} \\
&\qquad \times \Bigg[\bar{U}_c\hat{U}_c - (1-\delta_i)\beta\bar{\Omega}_i^r\mathbb{E}\Big[\bar{U}_c\hat{U}_c' + \frac{\bar{U}_d}{\bar{\psi}^d}(\hat{U}_d' - (\hat{\psi}^d)') + \Big(\bar{U}_c + \frac{\bar{U}_d}{\bar{\psi}^d}\Big)(\hat{\Omega}_i^r)'\Big]\Bigg]
\end{aligned}
$$

$$\hat{\Omega}_i^r = (1 - \beta(1-\delta_i))\hat{\psi}^x + (1 - \bar{\psi}^x)\beta(1-\delta_i)\mathbb{E}\Big[(\hat{\epsilon}_Q^{\psi^d})' + \hat{U}_c' - \hat{\epsilon}_Q^{\psi^d} - \hat{U}_c + (\Omega_i^r)'\Big]$$

$$\hat{I}' = -\bar{\psi}^x/(1-\bar{\psi}^x)\hat{\psi}^x + \hat{X}$$

$$\hat{N}' = \big(1 - \delta/(1-\bar{N})\big)\hat{N} + \delta(\hat{\theta} + \hat{\pi}^v)$$

$$\hat{y} = \hat{Y} - \hat{N}$$

$$\hat{\iota} = -1/(1-\bar{\psi}^x)\hat{\psi}^x$$

where $(\hat{Y}, \hat{C}, \hat{X}, \hat{F})$ are given by

$$
\begin{aligned}
\hat{Y} &= \hat{C} + (1-\delta_i)\frac{1-\bar{\psi}^x}{\bar{\psi}^x}(\hat{I}' - \hat{I}) \\
\hat{C} &= \hat{\psi}^x + \hat{X} \\
\hat{X} &= \big(1 - (1-\delta_i)(1-\bar{\psi}^x)\big)\hat{F} + (1-\delta_i)(1-\bar{\psi}^x)\hat{I} \\
\hat{F} &= \hat{z} + \frac{\lambda}{\bar{N} - \bar{\theta}(1-\bar{N})}((1+\bar{\theta})\bar{N}\hat{N} - (1-\bar{N})\bar{\theta}\hat{\theta})
\end{aligned}
$$

and in addition, given the choice of the functional forms

$$\hat{U}_c = \hat{\zeta}_c - \eta\hat{C} \qquad \hat{U}_d = \hat{\zeta}_d \qquad \hat{U}_n = \hat{\zeta}_n$$

$$\hat{\pi}^v = -\gamma\hat{\theta}$$

$$\hat{\psi}^x = -\frac{\alpha(\bar{Q}\bar{X})^\rho}{\alpha(\bar{Q}\bar{X})^\rho + 1 - \alpha}(\hat{Q} + \hat{X}) \qquad \hat{\epsilon}_Q^{\psi^d} = -\frac{\alpha(\bar{Q}\bar{X})^\rho}{\alpha(\bar{Q}\bar{X})^\rho + 1 - \alpha}\rho(\hat{Q} + \hat{X})$$

Appendix E

Data sources

Time series used in this paper were retrieved from the following sources:

1. Bureau of Economic Analysis (BEA) www.bea.gov
2. Bureau of Labor Statistics (BLS) www.bls.gov
3. NBER Macrohistory database (NBER) www.nber.org/databases/macrohistory/contents
4. Employment and Earnings data compiled by Cociuba, Prescott, and Ueberfeldt (2012)

In particular, following data was obtained.

National Income and Product Accounts (BEA:NIPA)

1. Table 1.7.5: Gross National Product GNP_t, Consumption of Fixed Capital DEP_t
2. Table 1.12: Compensation of Employees CE_t, Rental Income RI_t, Corporate Profits CP_t, Net Interests NI_t, Current Surplus of Government Enterprises GE_t
3. Table 1.1.4: Price Index for Gross Domestic Product $pGDP_t$

Fixed Assets Accounts Tables (BEA:FAA)

1. Table 1.1: Current Cost Net Private Fixed Assets K_{2005}
2. Table 1.2: Chain-Type Quantity Index for Private Fixed Assets qiK_t

Current Population Survey (BLS:CPS)

1. Civilian Noninstitutional Population, age 16 and more $P16_t$: Series ID LNU00000000
2. Civilian noninstitutional population, 65 years and over $P65_t$: Series ID LNU00000097
3. Employment E_t: Series ID LNU02005053
4. Average Weekly Hours AWH_t: Series ID LNU02005054

NBER Income and Employment (NBER:IE)

1. Average Weekly Hours AWH_t: Series m08354

Cociuba et al. (2012) (CPU)

1. Employment E_t
2. Average Weekly Hours AWH_t

Constructed time series

Labor share: obtained by constructing following time series

$$\lambda_t = 1 - \frac{RI_t + CP_t + NI_t + GE_t + DEP_t}{CE_t + RI_t + CP_t + NI_t + GE_t + DEP_t}$$

See Ríos-Rull and Santaeullia-Llopis (2010) for more details.
Hours: monthly time series for employment E_t and average weekly hours AWH_t were compiled from BLS:CPS, NBER:IE, and CPU sources described above, and were seasonally adjusted using Census X-12 Arima procedure with Easter and Labor day dummies. Then, quarterly averages were constructed, and total hours and hours per person of age 16-64 were obtained using $TH_t = E_t AWH_t$ and $H_t = TH_t/(P16_t - P64_t)$. Finally total hours were annualized and hours per person were expressed relative to 100 hours per week.
Real Capital K_t: obtained by multiplying the chain-type quantity index from BEA:FAA Table 1.2 by the current-cost net stock in 2005 from BEA:FAA Table 1.1, and interpolated to obtain quarterly time series.
Productivity residual: obtained by first taking a logarithm of GNP, real capital and total hours worked, then linearly detrending these time series and finally calculating

$$\log \hat{z}_t = \tilde{y}_t - (1 - \bar{\lambda})\tilde{k}_t - \bar{\lambda}\tilde{t}h_t$$

where $\bar{\lambda}$ is the average labor share, and for any variable X_t $\tilde{x}_t = \log X_t - a_X - b_X t$ is the residual from the linear detrending procedure.